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Effects of Damping On the Performance of Resonant Harmonic Filters

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Abstract: The effectiveness of resonant harmonic filters (RHFs) in harmonic suppression is the resultant of two different types of resonance that affect the filters' effectiveness in an opposite manner. They are the resonance of the filter branches and the resonance of the entire filter with the distribution system inductance. Damping these resonances by reduction of the filter Q-factor affects the filter performance in a complex way. There are suggestions in the literature on RHFs that such a damping would improve the filter effectiveness. Unfortunately, no quantitative information to support such a suggestion is available.

This paper presents the results from a study of the effect of the Q-factor on the filter effectiveness and on the loss of active power in the filter.

Key Words: resonant harmonic filters, harmonics

I. INTRODUCTION

In spite of the development and availability of active power filters for harmonic suppression, passive harmonic filters (PHSs) of various structures are still the main devices installed in distribution systems for reducing harmonic distortion. Although the technology of PHFs is a few decades old, there are still some open or controversial issues. Some of them are very complex, like the issue of localization of PHFs in a distribution system to optimize the system performance. Some of them are almost elementary. The question: "*how does the quality factor, $q = \omega L/R$, of the filter inductors effect the filter performance*," is just such an elementary issue.

Nonetheless, the opinions on the effect of the Q-factor on effectiveness of the basic PHSs it means, resonant harmonic filters (RHFs), are divided. According to Ref. [1], the Q-factor should be kept as high as possible, but a resistor that reduces the Q-factor should be added to the branch tuned to the 5th order harmonic when a resonance near the 4th harmonic frequency, namely in the band of $(3.85\text{-}4.15)\omega_1$, cannot be avoided. However, in order to damp the harmful resonances, connection of an additional resistor is recommended in Ref. [2]. According to Ref. [3] the filter "*resistance has a minor effect on harmonic attenuation*". With such information, not supported with a quantitative analysis, the filter designer might be confused. What would be the desirable level of the Q-factor?

The reason for this controversy is the fact that harmonics other than those the filter is tuned to, considered as minor, are sometimes [3, 4] neglected in the process of filter design. At such an assumption, the filter effectiveness increases with the inductor Q-factor increase without any doubts. However, some authors who discuss the design of filters and their performance based on measurements rather than on assumptions or simplifications, report [5-8] amplification of some minor harmonics rather than their attenuation by the filter. Indeed, it has been shown [9] that minor harmonics may contribute formidably to the reduction of the filter effectiveness. Amplification of these minor harmonics declines with the Q-factor decline and consequently, reduction in the Q-factor contributes to filter effectiveness increase. However, the resultant effect is difficult to predict without quantitative analysis.

The investigation of the impact of resonance damping on the effectiveness of RHFs and the power loss is the subject of this paper.

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It is easy to predict that no explicit results could be obtained for this kind of problem. It depends on the magnitude of minor harmonics, their origin and spectrum. Detailed results depend also on the system, the load and the filter parameters. This paper provides only a quantitative insight into the impact of resonance damping on the filter effectiveness and illustrates it with a simple case study. It may help the Reader to predict damping effects in other situations.

II. FUNDAMENTALS

Resonant harmonic filters (RHFs), installed at supply terminals of distribution systems, form a low impedance path for current harmonics generated in non-linear or time-variant loads, referred to as harmonic generating loads (HGLs), thus protecting the system against the injection of these harmonics. Such filters are built of a few LC branches tuned to different frequencies. Although other filter structures are also used, this is the structure used [10] most often.

The resistance, inductance and capacitance of a filter branch tuned to the frequency $\omega = z_n = 2\pi f_n$ are denoted in this paper by R_n, L_n and C_n. Since Q is used as the symbol for reactive power, the Q-factor of a branch at the tuning frequency, $\omega = z_n$, is denoted by q_n in this paper. It is equal to $q_n = z_n L_n/R_n$. According to Ref. [10], for high voltage applications where air-core inductors are used the Q-factors of $50 < q < 150$ are typical while for low voltage applications iron-core inductors are needed with $10 < q < 50$. The Q-factor of air-core inductors increases linearly with frequency. This increase is slower for iron-core inductors due to an increase of the power loss in the core.

The effectiveness of a filter branch in the protection of the system against the injection of a current harmonic to which the branch is tuned increases with reduction of the filter resistance R_n thus, with the filter branch Q-factor increase. Unfortunately, the conclusion that effectiveness of the filter increases as well, is correct only as long as there are no harmonics other than the load current harmonics to which the filter is tuned.

Harmonic generating loads usually also generate current harmonics other than those the filter is tuned to. Moreover, there are harmonics in the distribution voltage. These harmonics, considered as minor, often are neglected when the filter is designed.

The impedance of RHFs is capacitive in a band below each tuning frequency. Consequently, a resonance of the filter with the inductance of the distribution system has to occur in such a band. The number of these resonances is equal to the number of the filter branches. For example, a filter with branches tuned to the 5th, 7th, 11th and 13th harmonics has to have a resonance below the 5th order harmonic, between the 5th and 7th, between 7th and 11th and between 11th and 13th order harmonics.

As seen from the distribution system, these resonances are series or voltage resonances, thus the impedance from the supply side at such resonances has a minimum value. It increases the supply current harmonics, i_n, caused by distribution voltage harmonics, e_n, of the frequency in a vicinity of the resonant frequency. Moreover, the voltage resonance amplifies the bus voltage harmonics, u_n, in a vicinity of the resonant frequency.

As seen from the load, these resonances are parallel or current resonances, thus the impedance from the source of the load generated harmonics, Z_y, at the resonant frequency has a maximum value. It increases the load voltage harmonics, u_n, caused by the load generated current harmonics. Moreover, this resonance amplifies the supply current harmonics, i_n.

These four effects, caused by the resonance of the filter with distribution system inductance, contribute to an increase in the voltage and current distortion and thus they reduce the filter effectiveness. The filter Q-factor increase makes these harmful resonances and these effects more pronounced. Thus, the filter effectiveness in the presence of minor harmonics could be reduced with the Q-factor increase.

III. THE LOWEST RESONANT FREQUENCY

The equivalent circuit, per phase, of a distribution system, a harmonic generating load and a resonant harmonic filter composed of N-resonant branches has the form shown in Figure 1. The symbol $\boldsymbol{E}_n$ in this circuit stands for the complex RMS (CRMS) value of harmonics of the distribution voltage $e(t)$, the symbol $\boldsymbol{J}_n$ stands for the CRMS value of harmonics of the load generated current $j(t)$. Symbols $\boldsymbol{U}_\mathrm{n}$ and $\boldsymbol{I}_\mathrm{n}$ stand for the CRMS values of the load voltage, $u(t)$, and the supply current, $i(t)$, harmonics.

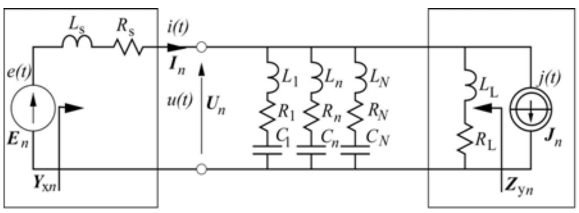

Fig. 1. Equivalent circuit

Symbol $\boldsymbol{Y}_\mathrm{xn}$ stands for the admittance as seen from the distribution system for the n^{th} order harmonic and the symbol $\boldsymbol{Z}_\mathrm{yn}$ stands for the impedance as seen from the source of load current harmonics. Moreover, let $U(j\omega)$ and $E(j\omega)$ denote the Fourier transforms of the load and distribution voltage, $u(t)$ and $e(t)$, and let $I(j\omega)$ and $J(j\omega)$ denote such transforms of the supply and generated currents, $i(t)$ and $j(t)$, respectively.

At tuning frequencies, $\omega = z_n$, the branch impedance approaches its minimum, which for a lossless filter equals to zero. The resonance with the distribution system occurs at the frequency, where the impedance as seen from the distribution system, $Z_\mathrm{x}(j\omega)$, has a minimum value. The system transmittances

$$A(j\omega) \;=\; \frac{U\,(j\omega)}{E\,(j\omega)}\,, \qquad B(j\omega) \;=\; \frac{I\,(j\omega)}{J\,(j\omega)}\,, \tag{1}$$

at such resonant frequency approach a maximum. For a lossless filter these transmittances approach infinity Frequencies of such resonances, $\omega = p_k$, are referred to as *poles*.

Let $Y_\mathrm{a}(j\omega)$ be the equivalent admittance of the RHF and the load, and $Z_\mathrm{s}(j\omega)$ is the distribution system impedance, namely

$$Y_\mathrm{a}(j\omega) = G_\mathrm{a}(\omega) + jB_\mathrm{a}(\omega) = Y_\mathrm{L}(j\omega) + \sum_{n=1}^{N} Y_n(j\omega)\;, \tag{2}$$

$$Z_\mathrm{s}\,(j\omega) \;=\; R_\mathrm{s}\,(\omega) + jX_\mathrm{s}\,(\omega)\;, \tag{3}$$

then, the system transmittances can be expressed as

$$A(j\omega) \;=\; B(j\omega) \;=\; \frac{1}{1 \;+\; Y_\mathrm{a}\,(j\omega)\,Z_\mathrm{s}\,(j\omega)}\,. \tag{4}$$

Case #1. Magnitudes of transmittances $A(j\omega)$ and $B(j\omega)$ calculated for three values of the Q-factor equal to q = 100, 30 and 10 are shown in Fig. 2. The plots are for a system with the load active power P = 50 kW at U = 480 V and PF of λ = 0.71, with two branch filter tuned to frequency $4.8\omega_1$ and $6.8\,\omega_1$, installed on a bus with the short circuit power S_sc = 2.0 MVA and $X_\mathrm{s}/R_\mathrm{s}$ = 5.

Figure 2 demonstrates the very explicit effect of the Q-factor on damping harmful resonances, and consequently, on the reduction of harmonic amplification. An increase of transmittances $A(j\omega)$ and $B(j\omega)$ at tuning frequencies with the Q-factor reduction is less visible but this can have substantial adverse impact on the filter effectiveeness. The frequency of the harmful resonances for the parameters assumed in Illustration 1 are above the frequency of the 4$^{\mathrm{th}}$ and very close to the 6$^{\mathrm{th}}$ order harmonics. However, some changes in the system parameters or filter detuning, may cause these resonances to occur much closer to or at harmonic frequencies.

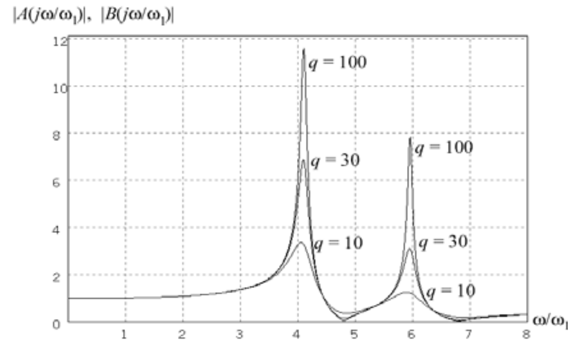

Fig. 2. Magnitues of transmittances $A(j\omega)$ and $B(j\omega)$

Harmful resonances are naturally damped by the load, supply and filter resistance, even without additional resistors in the filter branches.

Undamped resonances are resonances in the system with $G_\mathrm{a}(\omega)$ = 0 and $R_\mathrm{s}(\omega)$ = 0 and consequently, transmittances $A(j\omega)$ and $B(j\omega)$ at such resonances approach infinity. They occur at frequencies, $\omega = p_k$, where

$$Y_\mathrm{a}(jp_k)\,Z_\mathrm{s}(jp_k) = -\,B_\mathrm{a}(p_k)\,X_\mathrm{s}(p_k) = -\,1. \tag{5}$$

Unfortunately, the formula for calculating this frequency has a simple form only for single branch filters. In such a case, if the filter is tuned to frequency $\omega = z_1$, then the harmful resonance with the distribution system occurs at the frequency, $\omega = p_1$, equal to

$$p_1 \;=\; z_1\sqrt{\frac{1+L_\mathrm{s}/L_\mathrm{L}}{1+L_\mathrm{s}\,(1/L_\mathrm{L}+1/L_1)}}\,. \tag{6}$$

Usually $L_\mathrm{s} \ll L_\mathrm{L}$ thus, this formula could be simplified to

$$p_1 \;=\; \frac{z_1}{\sqrt{1+L_\mathrm{s}/L_1}}\,. \tag{7}$$

There are two harmful resonant frequencies, p_1 and p_2, for a two-branch filter. The formulae for calculating these frequencies are too complex, however, to have a practical meaning. The same is with higher number of branches. Numerical methods are needed.

Of all the resonances of the filter with distribution system, the lowest one is usually the most harmful, since its frequency may coincide with the frequency of the 3$^{\mathrm{rd}}$ or the 4$^{\mathrm{th}}$ order harmonics which usually have a higher value than other minor harmonics, such as the 6$^{\mathrm{th}}$, 8$^{\mathrm{th}}$ or the 9$^{\mathrm{th}}$ order. Also, as can be observed in Fig. 2, the lowest resonance is usually less damped than resonances at higher frequencies. Therefore, a formula for the resonant frequency, p_1, of the lowest harmful resonance would be desirable, even if it could provide only an approximate value of this frequency.

The first harmful resonance occurs below the lowest tuning frequency, z_1, that means at a frequency where the branch tuned to frequency, z_1, has much lower impedance than the remaining ones. Thus, the parameters of other branches could be neglected in the approximate formula for the first harmful resonance frequency, p_1.

If the filter branch, tuned to frequency z_1 compensates the reactive power equal to a_1Q, where a_1 denotes a *reactive power allocation coefficient*, then the branch should have the capacitance

$$C_1 = a_1\frac{Q}{\omega_1 U^2}\,[1 - (\frac{\omega_1}{z_1})^2] \approx a_1\frac{Q}{\omega_1 U^2} = a_1 C_0\;, \tag{8}$$

where C_0 denotes the capacitance of a compensator needed for the power factor improvement to unity.

$$C_0 = \frac{Q}{\omega_1 U^2} = \frac{P\ \tan\varphi}{\omega_1 U^2}\,. \tag{9}$$

The inductance of a branch is approximately equal to

$$L_1 = \frac{1}{a_1 z_1^2}\frac{1}{C_0}\,. \tag{10}$$

If the resistance of the distribution system, which is usually much lower than its reactance, is neglected, then its equivalent inductance can be found from the formula

$$L_\mathrm{s} = \frac{E^2}{\omega_1 S_\mathrm{sc}} \approx \frac{U^2}{\omega_1 S_\mathrm{sc}}\,. \tag{11}$$

Thus, taking into account formulae (9-11), formula (7) provides approximate value of the frequency of the lowest resonance

$$p_1 = \frac{z_1}{\sqrt{1 + a_1 (z_1/\omega_1)^2 \frac{P}{S_\mathrm{sc}} \tan\varphi}}\,. \tag{12}$$

This formula for the system with parameters as assumed in Case #1 results in the resonant frequency of the lowest resonance equal to p_1= 4.3 z_1. Taking into account that the system and filter parameters are not usually known with accuracy higher than 5-10 percent, it is difficult to expect that the resonant frequency could be calculated with high accuracy.

Formula (12) provides some information on how the resonant frequency could be controlled during the process of filter design. Filter detuning, i.e., a change of the z_1 with respect to harmonic frequency, $k\omega_1$, provides only very limited control of the resonant frequency, because this detuning usually cannot be greater than only a few percent. It should be observed, that detuning may shift the resonant frequency, p_1 toward a harmonic frequency. For example, detuning the 5th order branch to $z_1 = 4.7\omega_1$, would move the resonant frequency towards the 4th order harmonic frequency.

The coefficient, a_1, of the reactive power allocation provides a real tool for such a control. Unfortunately, any change in the distribution system that affects its inductance, L_s, may invalidate such a control.

IV. MAXIMUM HARMONIC AMPLIFICATION

Harmful resonances are damped by the resistance of the load, the resistance of the distribution system as observed at the bus where the filter is installed and by the resistance of the filter branches. Unfortunately, these resistances depend on frequency and their values are not easily available for the filter designer. Therefore, only a rough assessment of these resistances and the filter damping is possible. Such an assessment is presented below.

Distribution system resistance R_s and non zero conductance G_a of the filter and the load reduce the system transmittances, specified with formula (5), at resonant frequencies $A(jp_\mathrm{k})$ and $B(jp_\mathrm{k})$ from infinity to

$$A(jp_k) = B(jp_k) = \frac{1}{G_\mathrm{a} R_\mathrm{s} + j(G_\mathrm{a} X_\mathrm{s} +_\mathrm{s} B_\mathrm{a})}\,, \tag{13}$$

where parameters G_a, R_s, X_s and B_a are specified at resonant frequency, p_k. Usually $R_\mathrm{s} << X_\mathrm{s}$, thus, the real part of the denominator, $G_\mathrm{a}R_\mathrm{s}$, in the formula (13) can be neglected. Moreover, the equivalent conductance, G_a, is the sum of the filter conductance, G_F and the load conductance, G_L. Thus, the magnitude of transmittances $A(jp_k)$ and $B(jp_k)$, denoted as A_p and B_p, can be approximated as

$$A_\mathrm{p} = B_\mathrm{p} \approx \frac{1}{R_\mathrm{s} B_\mathrm{a} + G_\mathrm{L} X_\mathrm{s} + G_\mathrm{F} X_\mathrm{s}}\,. \tag{14}$$

The formula (14) can be rearranged as

$$A_\mathrm{p} = B_\mathrm{p} \approx A_\mathrm{p0} \frac{1}{1 + d_\mathrm{L} + d_\mathrm{F}}\,, \qquad A_\mathrm{p0} = B_\mathrm{p0} = \frac{1}{R_\mathrm{s} B_\mathrm{a}}\,. \tag{15}$$

The symbol A_p0 denotes harmonic amplification at a resonant frequency in the lack of resonance damping by the load and the filter resistance, i.e., when $G_\mathrm{F} = 0$ and $G_\mathrm{L} = 0$. The term $A_\mathrm{p0} = B_\mathrm{p0}$ specifies maximum harmonic amplification possible at the bus when the load is purely reactive and the filter has an infinite Q-factor. The coefficient

$$d_\mathrm{L} = \frac{G_\mathrm{L}}{B_\mathrm{a}}\xi_\mathrm{s}, \qquad \xi_\mathrm{s} = \frac{X_\mathrm{s}}{R_\mathrm{s}}\,, \tag{16}$$

specifies a coefficient of resonance damping due to the load resistance, while the coefficient

$$d_\mathrm{F} = \frac{G_\mathrm{F}}{B_\mathrm{a}}\xi_\mathrm{s}\,, \tag{17}$$

specifies a coefficient of resonance damping due to the filter resistance.

The maximum possible harmonic amplification A_p0 at a resonant frequency depends on the equivalent resistance of the distribution system R_s at frequency p_k. For a rough approximation, it can be assumed that its value is the same as for the fundamental harmonic and can be calculated from the formula

$$R_\mathrm{s} \approx \frac{U^2}{S_\mathrm{sc}\sqrt{1+\xi_\mathrm{s}^2}}\,, \tag{18}$$

The equivalent susceptance B_a is a sum of the filter and load susceptances,

$$B_\mathrm{a} = B_\mathrm{F} + B_\mathrm{L}, \tag{19}$$

and their values have to be analysed separately.

The load susceptance at frequency $\omega = p_k$ is equal to

$$B_\mathrm{L} = -\frac{1}{R_\mathrm{L}} \frac{\Omega_k \tan\varphi}{1 + \Omega_k^2 \tan^2\varphi}\,. \tag{20}$$

where $R_\mathrm{L} = U^2\lambda^2/P$, $\varphi = \cos^{-1}(\lambda)$ and $\Omega_k = p_\mathrm{k}/\omega_1$. For $p_\mathrm{k} >> \omega_1$ and common values of the power factor λ, the load susceptance can be approximated by

$$B_\mathrm{L} = -\frac{1}{R_\mathrm{L}\,\Omega_k \tan\varphi} = -\frac{1}{\Omega_k}\frac{P}{\lambda\sqrt{1-\lambda^2}\,U^2}\,. \tag{21}$$

thus, its magnitude declines monotonically with the increase of the relative resonant frequency $\Omega_\mathrm{k} = p_\mathrm{k}/\omega_1$.

The filter susceptance, B_F, at resonant frequency, p_k, is equal to the sum of the branch susceptances, B_n. The effect of the branch resistance, R_n, on the branch susceptance is usually negligible, thus the filter susceptance can be approximated as

$$B_\mathrm{F} = \sum_{n=1}^{n=N} B_n \approx \sum_{n=1}^{n=N} \frac{p_k C_n}{1 - p_k^2 L_n C_n} = \sum_{n=1}^{n=N} \frac{p_k C_n}{1 - (p_k/z_n)^2}\,, \tag{22}$$

If the n-branch of the filter compensates the reactive power $a_n Q$ and the filter is tuned to frequency z_n, then

$$\omega_1 C_n = a_n \frac{Q}{U^2}[1 - (\omega_1/z_n)^2]\,. \tag{23}$$

Thus, taking into account formula (23), the filter susceptance at frequency p_k can be expressed as

$$B_\mathrm{F} \approx \frac{Q}{U^2}\Omega_k \sum_{n=1}^{N} a_n \frac{\zeta_n^2 - 1}{\zeta_n^2 - \Omega_k^2}\,, \quad \text{with} \quad \zeta_n = \frac{z_n}{\omega_1}\,. \tag{24}$$

Case #2. This case illustrates the filter resonance without damping by the load, i.e., in a system with a purely reactive load. The apparent power at the voltage U = 1pu is assumed as the power reference. The filter, installed at the bus with the short circuit power S_sc = 40 pu and $X_\mathrm{s}/R_\mathrm{s}$ = 5, is tuned to 4.7ω_1 and 6.7 ω_1. The resonant frequencies are approximately equal to p_1 = 4.05 ω_1 and p_2 = 5.9 ω_1. The load resistance, calculated from formula (18), is equal to R_s = 0.0065 pu. The load and the filter susceptance are compiled in Table 1. The amplification at resonant frequencies at the lack of damping by the load and the filter resistance at frequency p_1 = 4.05 ω_1, according to formula (17), is equal to A_p0 = 19 and at frequency p_2 = 5.9 ω_1 it is equal to A_p0 = 29. These amplifications obtained from the system model are 20 and 28, respectively.

Table 1. Load and filter susceptances

p_k	rd/s	4.05 ω_1	5.9 ω_1
B_L	pu	- 0.25	- 0.16
B_1	pu	5.19	- 6.26
B_2	pu	2.89	11.71
B_F	pu	8.08	5.43

Thus, in spite of the assumed approximations, these results differ only by a few percent from the results obtained from system modeling, shown in Fig. 3.

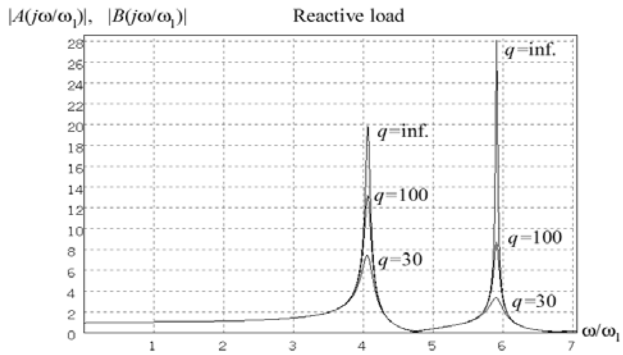

Fig. 3. Magnitude of transmittances $A(j\omega)$ and $B(j\omega)$ for reactive load

The values compiled in this Table show that the load susceptance has relatively low effect on the maximum harmonic amplification.

V. RESONANCE DAMPING

If it is assumed that equivalent resistance of the distribution system R_s does not change with frequency, which is only an approximation, i.e., the reactance to resistance ratio $\xi_s = X_s/R_s$ increases from its value at the fundamental, ξ_{s1}, the load related damping coefficient, d_L, specified with formula (16), can be expressed as follows

$$d_L = \frac{X_s}{R_s}\frac{G_L}{B_a} = \Omega_k \xi_{s1}\frac{G_L}{B_a}, \tag{25}$$

where the load conductance at resonant frequency, p_k, can be approximated by

$$G_L = \frac{1}{R_L}\frac{1}{1+\Omega_k^2\tan^2\varphi} \approx \frac{1}{\Omega_k^2}\frac{P}{U^2\sin^2\varphi}. \tag{26}$$

The filter related damping coefficient, d_F, specified with formula (17) can be expressed as

$$d_F = \Omega_k\xi_{s1}\frac{G_F}{B_a} = \Omega_k\xi_{s1}\frac{1}{B_a}\sum_{n=1}^{n=N} G_n, \tag{27}$$

where the conductance at frequency p_k of the filter n^{th}-branch tuned to frequency z_n is equal to

$$G_n = \frac{1}{R_n}\frac{(p_kR_nC_n)^2}{[1-(z_n/p_k)^2]^2+(p_kR_nC_n)^2}. \tag{28}$$

Since

$$p_kR_nC_n = (p_k/z_n)\frac{1}{q_n} << 1-(p_k/z_n)^2, \tag{29}$$

the conductance G_n is approximately equal to

$$G_n \approx \frac{1}{(z_n/p_k - p_k/z_n)^2}\frac{z_nC_n}{q_n}, \tag{30}$$

where q_n is the branch Q-factor at the tuning frequency,

$$q_n = \frac{z_nL_n}{R_n}. \tag{31}$$

The conductance of the filter n-branch can be expressed in terms of the branch reactive power. It is equal to

$$G_n \approx a_n\frac{1}{q_n}\frac{z_n/\omega_1-\omega_1/z_n}{(z_n/p_k-p_k/z_n)^2}\frac{Q}{U^2}. \tag{32}$$

Formulae for load and filter damping coefficients (25) and (27) show that the load and filter resonance damping coefficients remain independent on the supply short-circuit power, S_{sc}, which affects the resonant frequencies, p_k. The most crucial situation occurs when the resonant frequency coincides with the 4^{th} or with the 6^{th} order harmonics. Only in very weak systems with a low load power factor would this frequency coincide with the 3^{rd} order harmonic. Therefore, the evaluation of the harmonic amplification, thus, its maximum value and damping coefficients, for the 4^{th} and 6^{th} order harmonics is particularly important.

The values of these damping coefficients along with the system parameters for the situation in Case 1 for two different Q-factors, the same for each branch, q = 100 and 30, are compiled in Table 2.

Table 2. Load and the filter parameters at resonant frequency

p_k	rd/s	4.0 ω_1		6.0 ω_1	
G_L	pu	0.125		0.041	
B_L	pu	- 0.50		- 0.286	
B_1	pu	5.33		- 6.54	
B_2	pu	2.91		11.08	
B_a	pu	7.74		4.25	
d_L	-	0.32		0.29	
q_n	-	100	30	100	30
G_1	pu	0.119	0.396	0.179	0.596
G_2	pu	0.025	0.083	0.358	1.19
G_F	pu	0.144	0.479	0.537	1.79
d_F	-	0.37	1.24	3.79	12.6

The results compiled in Table 2 show that for a common level of the filter Q-factor, q, the load resistance has much lower contribution to the resonance damping than the filter resistance. Also, these results show that the lowest resonance is much less damped than the resonance at higher frequencies. This confirms the earlier conclusion, that the lowest resonance, as less attenuated is much more crucial for the filter performance than resonances at higher frequencies, especially since this usually relates to the 4^{th} and the 6^{th} order harmonic and the first of them is usually much larger.

Observe that according to formulae (15, 16 and 17), both the maximum harmonic amplification A_{p0} and the filter resonance damping coefficients depend on the equivalent susceptance, B_a. However, when

$$d_F >> 1 + d_L, \tag{33}$$

the harmonic amplification at resonant frequency, p_k, can be approximated by

$$A_p \approx \frac{A_{p0}}{d_F} = \frac{1}{\Omega_k R_s\xi_{s1}G_F} = \frac{1}{\Omega_k X_s G_F}, \tag{34}$$

thus, it is independent of the filter and the load susceptance.

VI. ADMITTANCE FOR DISTRIBUTION HARMONICS

The resonance of the filter with the distribution system causes not only resonant amplification of the voltage and current harmonics. This resonance also increases the admittance as seen from the distribution system, defined as

$$Y_x(j\omega) = \frac{I(j\omega)}{E(j\omega)} = \frac{Y_a(j\omega)}{1+Y_a(j\omega)Z_s(j\omega)}. \tag{35}$$

At resonant frequencies this admittance may approach a very high value. Consequently, if the resonant frequency coincides with the frequency of a distribution voltage harmonic, a large harmonic can occur in the supply current. Moreover, this admittance has a high value for the voltage harmonics to which the filter is tuned. An example of the plot of the magnitude of admittance $Y_x(j\omega)$ is shown in Fig. 4. This plot was drawn for a system with the same parameters as in Case 1 and for three values of the Q-factor, namely, $q = \infty$, q = 100 and q = 30, respectively.

In the lack of the resonance damping by the filter and load resistance, the magnitude of admittance $Y_x(j\omega)$ at the resonant frequency,

p_k, has a maximum value, denoted by Y_{xp0}. The resonant current is bounded only by the distribution system resistance, R_s, hence

$$Y_{xp0} = 1/R_s \,. \quad (36)$$

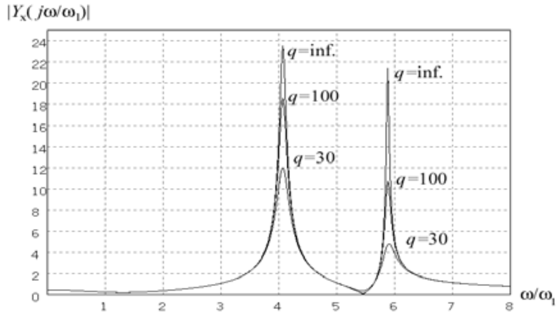

Fig. 4. Plot of magnitude of admittance $Y_x(j\omega)$ versus frequency

The effect of the load and the filter resistance on the resonance admittance, Y_{xp}, can be evaluated, assuming that the equivalent conductance G_n is much lower than susceptance B_n, using a formula which is similar to formula (15) for the resonant harmonic amplification,

$$Y_{xp} \approx Y_{xp0}\,\frac{1}{1+d_L+d_F}\,, \quad (37)$$

where coefficients d_L and d_F are defined by (16) and (17).

VII. IMPEDANCE FOR LOAD HARMONICS

The resonant harmonic filter changes not only the supply current at the bus where the filter is installed, but also the impedance as seen from the harmonic generating load. Consequently, it affects the bus voltage harmonics caused by the load current harmonics. The impedance for these harmonics is equal to

$$Z_y(j\omega) = \frac{U(j\omega)}{J(j\omega)} = \frac{Z_s(j\omega)}{1+Y_a(j\omega)\,Z_s(j\omega)}\,. \quad (38)$$

This impedance, drawn for a system with the same parameters as in Case 1, is shown in Fig. 5.

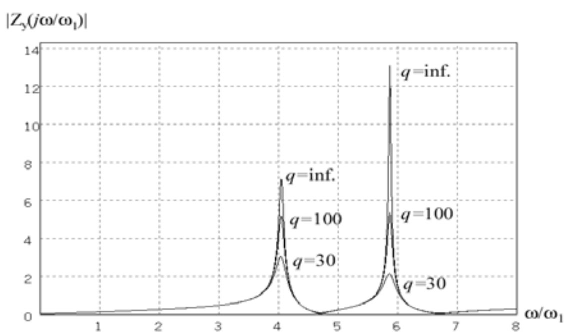

Fig. 5. Plot of magnitude of impedance $Z_y(j\omega)$ versus frequency

At the resonant frequency the impedance $Z_y(jp_k) = Z_{yp}$ is equal to

$$\boldsymbol{Z}_{yp} = \frac{R_s + jX_s}{G_a R_s + j\,(G_a X_s + R_s B_a)}\,. \quad (39)$$

Typically, $R_s << X_s$, thus, this impedance can be approximated by

$$Z_{yp} \approx Z_{yp0}\,\frac{1}{1+d_L+d_F}\,, \qquad Z_{yp0} = \frac{X_s}{R_s B_a} = \Omega_k\,\frac{\xi_{s1}}{B_a}\,. \quad (40)$$

The impedance Z_{yp0} and consequently, the bus voltage distortion caused by the load generated current harmonics j_n increases with the resonance frequency increase and with increase in the reactance to resistance ratio ξ_{s1}. It also depends on the susceptance B_a.

VIII. DISTORTION AND POWER LOSS

Load generated current harmonics j_n and distribution voltage harmonics, e_n, are not mutually correlated, therefore, the expected RMS value, I_n, of the supply current harmonics can be calculated as the root of the sum of squares of RMS values of the load and the supply originated harmonics,

$$I_n = \sqrt{(B_n J_n)^2 + (Y_{xn} E_n)^2}\,, \quad (41)$$

where B_n and Y_{xn} denote the magnitudes of $B(j\omega)$ and $Y_x(j\omega)$ for the n-order harmonic. The supply current distortion, δ_i, which is the ratio of the RMS value $||i_d||$ of the distorting component of the supply current to the fundamental current RMS value, I_1, is equal to

$$\delta_i = \frac{||i_d||}{I_1} = \sqrt{\sum_{n=2}^{\infty}\frac{(B_n J_n)^2}{I_1^2} + \sum_{n=2}^{\infty}\frac{(Y_{xn} E_n)^2}{I_1^2}} = \sqrt{\delta_i^2(j) + \delta_i^2(e)}\,, \quad (42)$$

where $\delta_i(j)$ and $\delta_i(e)$ are coefficients of the supply current distortion caused by the load generated current harmonics and the distribution voltage harmonics. Similarly, the bus voltage distortion is equal to

$$\delta_u = \frac{||u_d||}{U_1} = \sqrt{\sum_{n=2}^{\infty}\frac{(A_n E_n)^2}{U_1^2} + \sum_{n=2}^{\infty}\frac{(Z_{yn} J_n)^2}{U_1^2}} = \sqrt{\delta_u^2(e) + \delta_u^2(j)}\,, \quad (43)$$

where $||u_d||$ is the RMS value of the distorting component of the bus voltage; A_n and Z_{yn} denote the magnitudes of $A(j\omega)$ and $Z_y(j\omega)$ for the n-order harmonic and $\delta_u(e)$ and $\delta_u(j)$ denote coefficients of the bus voltage distortion caused by the distribution voltage harmonics and by the load generated current harmonics, respectively.

The idea of reducing the voltage and current distortion due to the filter resonance by resonance damping is an old one. It was suggested by Steeper and Stradford, [1], as well as being reported in Ref. [2] by Merhaj and Nichols.

Reduction of the Q-factor, reduces the magnitude of transmittances $A(j\omega)$, $B(j\omega)$, $Y_x(j\omega)$ and $Z_y(j\omega)$ at resonant frequencies, p_k. At the same time, it increases the magnitude of these transmittances at tuning frequencies, z_k. Consequently, the reduction of the filter Q-factor affects the suppression of minor harmonics and the harmonics to which the filter is tuned in an opposite way. The resultant effect depends on the voltage and current spectra and on the frequencies of the filter resonance with the distribution system. If such a resonance coincides with the frequency of a minor harmonic, the reduction of the Q-factor may reduce the waveform distortion. Otherwise, the distortion may increase due to lower efficiency in attenuation of harmonics to which the filter is tuned.

In the case of filters of the 5th, 7th, 11th and 13th order harmonics the most crucial for the filter performance is a possible resonance at the frequency of the 4th order harmonic. This is the less attenuated resonance and the 4th order harmonic is usually stronger then the 6th, 8th and other non-characteristic harmonics.

The resonance curves, as it can be seen in Figs. 2-5, are very narrow. Thus, a resonance in the middle between frequencies of the 3rd and 4th order harmonic does not magnify the 4th order harmonic in the supply current and the bus voltage. There is no reason to be concerned with such a resonance. However, a reduction in the filter capacitance or the distribution inductance on the order of 30 per cent shifts this resonance from $3.5\omega_1$ to $4\omega_1$. Such a shift could be caused by a reconfiguration in the distribution system or by disconnection of a section of the capacitor bank, for example, by over-current protection circuits. This resonance in the presence of the 4th order harmonic in the distribution voltage or in the load current, even well below 1 per cent, may cause drastic increase of the bus voltage and the supply current distortion. Such distortion, even over a very short time, could disturb control systems of industrial processes. Therefore, the possibility of such distortion, its magnitude and means of prevention could be a matter of concern. The coincidence of the resonance at the 4th and 6th order harmonic, although possible is not probable. Also, this could be prevented at the filter design level.

Variation of particular components of the bus voltage and the supply current distortion, $\delta_i(e)$, $\delta_i(j)$, $\delta_u(e)$ and $\delta_u(j)$, with the Q-Factor, q, in the situation of Case 1, in the presence of only the 4th order harmonic in the distribution voltage, with E_4 = 0.5 %, is shown in Fig. 6. The load generated current does not contain any other harmonic but only J_5 = 18% and J_7 = 11% of the current fundamental harmonic.

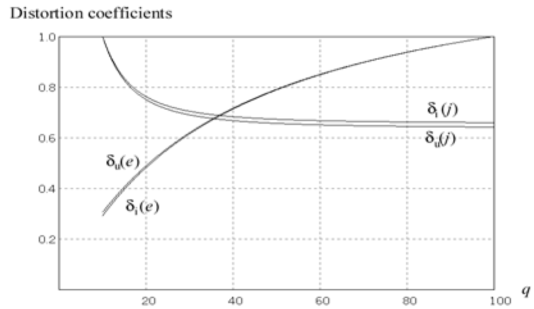

Fig. 6. Partial distortion coefficients versus Q-Factor, *q*

The level of 1.0 in Fig. 6 is the level of maximum values of particular distortion coefficients in the Q-factor variation range of (10-100). These maxima are equal to

$$\delta_i(e) = 49.7\%,\quad \delta_i(j) = 8.8\%,\quad \delta_u(e) = 5.7\%,\quad \delta_u(j) = 1.3\%.$$

The distortion caused by the load generated characteristic 5th and 7th order harmonics and the distribution voltage 4th order harmonics change with the Q-Factor in an opposite way, thus the respective maxima are at the opposite ends of the range of the Q-Factor variation. Distortion caused by the supply voltage 4th order harmonic is much higher than that caused by the load current. Thus, the minimum resultant voltage and current distortion, calculated according to formulae (42) and (43) occurs at the lower end of the Q-Factor variation range, i.e., at q = 10. It is approximately equal to

$$\delta_i \approx \sqrt{(0.3\times 49.7)^2 + 8.8^2} = 17.3\%\ ,$$

$$\delta_u \approx \sqrt{(0.3\times 5.7)^2 + 1.3^2} = 2.1\%\ .$$

In the case considered, the 4th order distribution voltage harmonic, e_4, is the major cause of waveform distortion. Observe, that the IEEE Standard 519 accepts only 25% of the even order harmonics in the bus voltage. Thus, even at the minimum value of voltage distortion, the contents of the 4th harmonic in the bus voltage, equal to 0.3 x 5.7 = 1.7%, exceeds this level.

Reduction in the Q-Factor of the filter branches is accompanied with an increase in the active power loss, ΔP_F, in the filter. The change of this power loss with the Q-Factor for the Case 1, along with change of total distortion coefficients of the bus voltage, δ_u and supply current, δ_i, are shown in Fig. 7.

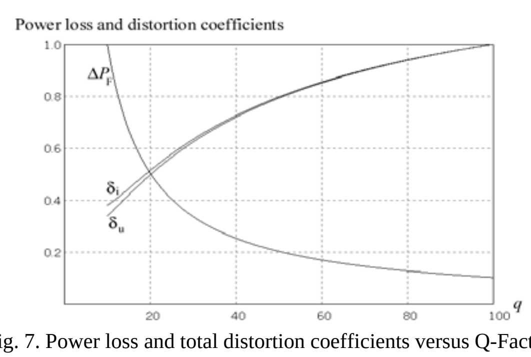

Fig. 7. Power loss and total distortion coefficients versus Q-Factor, *q*

The level of 1.0 in this Figure is the level of maximum power loss ΔP_F = 2.0% of the load active power and maximum values of the total distortion coefficients in the Q-factor variation range of (10-100), equal to δ_i = 50.0% and δ_u = 5.7%.

The results demonstrated above show that resonant amplification of the 4th order harmonic could be only reduced at the cost of substantial reduction of the filter Q-Factor and active power loss increase. The presence of the 4th order harmonic in the load generated current, usually caused usually by an asymmetry of AC/DC converters or by a jitter of thyfistors' firing angle, could aggravate the problem of the 4th order harmonic amplification even further. The effect of this harmonic of the value of J_4 = 1% on distortion coefficients is shown in Fig. 8.

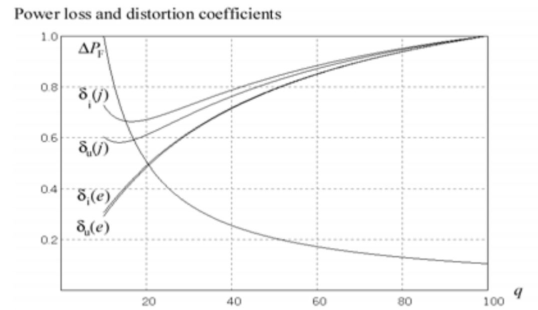

Fig. 8. Power loss and partial distortion coefficients versus Q-Factor, *q*

The maximum values in this plot are equal to

ΔP_F = 2.0%, $\delta_i(e)$ = 49.7%, $\delta_i(j)$ = 16.6%, $\delta_u(e)$ = 5.7%, $\delta_u(j)$ = 2.0%.

IX. CONCLUSIONS

Both analysis of resonance damping and case investigation show that if resonant harmonic filters tuned to the 5th and 7th order harmonics are installed at the bus with the short circuit power in the range of 30-50 times the active power at the bus, then the most harmful resonance of the filter with the distribution system can occur in the vicinity of the 4th order harmonic. A change in the system short circuit power or in the filter capacitance in the range of 30% can cause a coincidence of this resonance with the 4th order harmonic and its drastic amplification. In spite of suggestions in the literature that such a resonance could be damped by reduction in the filter Q-Factor, the results of the study in this paper do not confirm them. In order to effectively damp this resonance, the Q-Factor has to be reduced to such low level, that the cost of the active power loss in the filter could be unacceptable. It seems that a system of detection of resonance conditions and a method of prevention that this resonance does not approach the 4th order harmonic frequency would be more recommended than damping.

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