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On Some Misinterpretations of the Instantaneous Reactive Power p-q Theory

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Abstract. **The main features of the Instantaneous Reactive Power (IRP) p-q Theory, considered as a power theory of three-phase systems, are analyzed in this paper using the Theory of the Currents' Physical Components (CPC). This analysis shows that the *p* and *q* powers are not associated with separate power phenomena, but with multiple phenomena. Moreover, the results of the IRP p-q Theory contradict some common interpretations of power phenomena in three-phase circuits. Namely, according to the IRP p-q Theory the instantaneous reactive current can occur even if a load has zero reactive power, *Q*. Similarly, the instantaneous active current can occur even if a load has zero active power, *P*. Moreover, these two currents in circuits with a sinusoidal supply voltage can be nonsinusoidal even if there is no source of current distortion in the load. The analysis shows that a pair of values of instantaneous active and reactive *p* and *q* powers does not enable us to draw any conclusion with respect to the power properties of three-phase unbalanced loads even in a sinusoidal situation. Thus, the Instantaneous Reactive Power p-q Theory does not identify power properties of such loads instantaneously. This conclusion may have an importance for control algorithms of active power filters. The paper reveals the relationship between the *p* and *q* powers and the active, reactive and unbalanced powers, *P*, *Q* and *D* and specifies the required energy storage capability of active power filters operated under sinusoidal unbalanced conditions.**

Index Terms **– Instantaneous reactive power, instantaneous active power, reactive current, active current, unbalanced systems, apparent power, currents' physical components (CPC) power theory, unbalanced power, active power filters, switching compensators.**

1. Introduction

The Instantaneous Reactive Power (IRP) p-q Theory, developed by Akagi, Kanazawa and Nabae, [1,2], provides mathematical fundamentals for the control of switching compensators, known commonly as "active power filters".

Although there is still substantial confusion [3] with respect to power phenomena in electrical systems and there are reports [4, 5] on some shortcomings of the IRP p-q Theory, it seems to be well established [6-9] in the electrical engineering community involved in switching compensator design. Moreover, there are attempts [10-12] to provide deeper fundamentals for this Theory and it is becoming a theoretical tool for power properties of three-phase system analysis [13-15] and instrumentation [16].

When the IRP p-q Theory is considered as a theoretical fundamental for a control algorithm design, it is irrelevant whether it interprets power properties of electrical circuits correctly or not. It is enough that it enables the achievement of the control objectives. However, when it is considered as a power theory one could expect that it does provide a credible interpretation of power phenomena in electrical systems.

Having this expectation in mind, the following dilemma occurs. Power properties of three-phase, three-wire systems with only sinusoidal voltages and currents, i.e., even without any harmonic distortion, are determined by three *independent features of the system.* (i) permanent energy transmission and associated active power, *P*, (ii) presence of reactive elements in the load and associated reactive power, *Q*, and (iii) load imbalance that causes supply current asymmetry and associated unbalanced power, *D*. Thus, how can the IRP p-q Theory, based on only two power quantities, *p* and *q*, identify and describe three independent power properties? Moreover, according to Akagi and Nabae, [2] who developed the Instantaneous Reactive Power p-q Theory, its development was a response to "*...the demand to instantaneously compensate the reactive power...*" The adverb *instantaneous* in the name of this Theory and definitions of *p* and *q* powers in terms of instantaneous value of voltages and currents, suggest the possibility of instantaneous identification and compensation of the reactive power of a three-phase load. This is one of the main reasons for this theory's attractiveness, both as a theoretical fundamental of control algorithms and as a power theory. Thus, the question occurs, is such an instantaneous identification of power properties of three-phase systems possible? The answer to this question is of a fundamental value for the power theory. It could be important also for control algorithms of active power filters.

To answer such a question, this paper investigates how the IRP p-q Theory interprets and describes power phenomena in three-phase, three-wire systems. The Theory of the Currents' Physical Components (CPC) developed in Ref. [17] by the author of this paper is used as a tool for the study.

The IRP p-q Theory was developed for three-phase systems with nonsinusoidal voltages and currents. To provide credible results for such systems, it should provide them for any sub-set of such systems. Three-phase, three-wire systems with sinusoidal voltages and currents form just such a sub-set with relatively simple and easy to comprehend power phenomena. Therefore, properties of the IRP p-q Theory are verified using such simple circuits.

2. Instantaneous Active and Reactive Currents

The notions of the active and reactive currents have meanings that were established in electrical engineering long ago. The active current, defined by Fryze [18] in 1932, is the smallest load current that is necessary [10] if the load at the supply voltage $u(t)$ has the active power, P, and has the same waveform as the supply voltage. This current was defined as

$$i_a(t) = \frac{P}{U^2} u(t) = G_e\, u(t), \tag{1}$$

where U is the supply voltage RMS value.

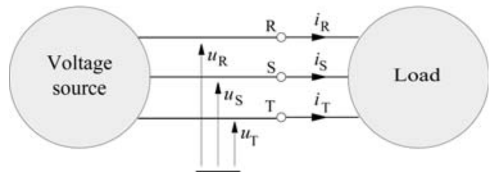

Fig. 1.

For three-phase, three-wire systems, shown in Fig. 1, Fryze's definition of the active current is generalized [17] to the form

$$\boldsymbol{i}_a(t) = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \frac{P}{\|\boldsymbol{u}\|^2} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = G_e\, \boldsymbol{u}(t), \tag{2}$$

where $\|u\|$ denotes the three-phase RMS value of the supply voltage, namely

$$\|\boldsymbol{u}\| = \sqrt{U_R^2 + U_S^2 + U_T^2}\,. \tag{3}$$

The reactive current is the component of the supply current delayed by $\pi/2$ with respect to the supply voltage and is defined in single-phase systems with sinusoidal voltage and current as

$$i_r(t) = \frac{-Q}{U^2} \frac{d}{d(\omega t)} u(t) = B_e \frac{d}{d(\omega t)} u(t)\,. \tag{4}$$

For three-phase, three-wire systems this definition is generalized [17, 21] to

$$\boldsymbol{i}_r(t) = \frac{-Q}{\|\boldsymbol{u}\|^2} \frac{d}{d(\omega t)} \boldsymbol{u}(t) = B_e \frac{d}{d(\omega t)} \boldsymbol{u}(t)\,. \tag{5}$$

Both the active and reactive currents have an explicit physical meaning. They are associated with the presence of the active and reactive powers, P and Q, and are related to the load equivalent conductance, G_e, and susceptance, B_e. The concept of the active current is also important for the design of compensators. Because it is the smallest supply current of the load that has the active power, P, this is the only current that should remain in the supply lines of the load after compensation of all useless current components.

Now, let us compare the features of the active and reactive currents, as defined by (2) and (5), with the instantaneous active and reactive currents in the IRP p-q Theory. Observe however, that the adjective *instantaneous* in the name of these currents does not distinguish them from the common active and reactive currents, since definitions (2) and (5) specify instantaneous values of these currents.

The IRP p-q Theory has evolved from the Fortescue, Park and Clarke Transforms of voltages and currents specified in natural, phase R, S and T coordinates, as shown in Figure 1, into a pair of voltages and currents in orthogonal α and β coordinates. The Clarke Transform of three-phase voltages has the form

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix}. \tag{6}$$

For three-phase, three-wire systems as shown in Figure 1, with line voltages referenced to an artificial zero, so that $u_R + u_S + u_T \equiv 0$, the Clarke Transform of the line voltages can be simplified to the form

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} u_R \\ u_S \end{bmatrix} = \mathbf{C} \begin{bmatrix} u_R \\ u_S \end{bmatrix}, \tag{7}$$

and similarly for the line currents, since $i_R + i_S + i_T \equiv 0$,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3/2}, & 0 \\ 1/\sqrt{2}, & \sqrt{2} \end{bmatrix} \begin{bmatrix} i_R \\ i_S \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ i_S \end{bmatrix} . \tag{8}$$

The line currents can be calculated from the currents in the α and β coordinates with the inverse Clarke Transform

$$\begin{bmatrix} i_R \\ i_S \end{bmatrix} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} . \tag{9}$$

With the voltages and currents transformed to the α and β coordinates, the instantaneous power of the load can be expressed as

$$p = u_\alpha i_\alpha + u_\beta i_\beta , \tag{10}$$

referred to as the *instantaneous real* or *instantaneous active power* in the IRP p-q Theory. The *instantaneous imaginary power*, q, is defined in Ref. [2] as

$$q = u_\alpha i_\beta - u_\beta i_\alpha , \tag{11}$$

and is usually referred to [6-16] as the *instantaneous reactive power*.

With these two instantaneous powers, the instantaneous active and reactive currents are defined in Ref. [2]. The *instantaneous active current*, i_p, is defined in the α and β coordinates as

$$i_{\alpha p} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\, p , \qquad i_{\beta p} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2}\, p , \tag{12}$$

while the *instantaneous reactive current*, i_q, is defined as

$$i_{\alpha q} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2}\, q , \qquad i_{\beta q} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\, q . \tag{13}$$

The instantaneous active and reactive currents in the supply lines can be calculated from these currents in the α and β coordinates with the inverse Clarke Transform, (9), namely

$$\begin{bmatrix} i_{Rp} \\ i_{Sp} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix}, \qquad \begin{bmatrix} i_{Rq} \\ i_{Sq} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix}. \tag{14}$$

Unfortunately, the reactive current has little in common with the reactive power, Q, of the load. This is shown in Illustration 1, where the IRP p-q Theory is applied to a circuit with zero reactive power, Q.

Illustration 1. Let us assume that a resistive load, connected as shown in Fig. 2, is supplied from a symmetrical source of a sinusoidal, positive sequence voltage, with $u_R = \sqrt{2}\, U \cos \omega_1 t$, $U = 120$ V.

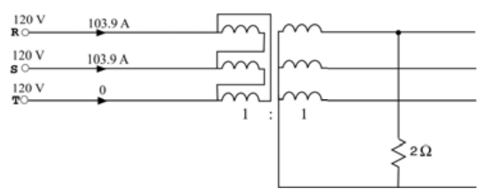

Fig. 2.

Given such assumptions, the supply voltage in the α and β coordinates has the value

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}\, U \cos\omega_1 t \\ \sqrt{2}\, U \cos(\omega_1 t - 120^o) \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, U \cos\omega_1 t \\ \sqrt{3}\, U \sin\omega_1 t \end{bmatrix} . \tag{15}$$

Since the line currents are equal to

$$i_R = \sqrt{2}\, I \cos(\omega_1 t + 30^0) = -\, i_S, \qquad I = 103.9 \text{ A}, \qquad i_T = 0.$$

The supply current in the α and β coordinates has the value

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ -\, i_R \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, I \cos(\omega_1 t + 30^o) \\ -I \cos(\omega_1 t + 30^o) \end{bmatrix} , \tag{16}$$

and consequently, the instantaneous active power of such a load is equal to

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\, U I [1 + \cos 2(\omega_1 t + 30^0)] , \tag{17}$$

and the instantaneous reactive power is equal to

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\,U I \sin 2(\omega_1 t + 30^0)\,. \tag{18}$$

Thus, the instantaneous active current of the load in the α and β coordinates, can be calculated. It is equal to

$$i_{\alpha\mathrm{p}} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\,p = I\,[1 + \cos 2(\omega_1 t + 30^0)]\cos\omega_1 t\,, \tag{19}$$

$$i_{\beta\mathrm{p}} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2}\,p = I\,[1 + \cos 2(\omega_1 t + 30^0)]\sin\omega_1 t\,, \tag{20}$$

which in the phase coordinates is equal to

$$\begin{bmatrix} i_{\mathrm{Rp}} \\ i_{\mathrm{Sp}} \end{bmatrix} = I\,[1 + \cos 2(\omega_1 t + 30^0)]\,\mathbf{C}^{-1}\begin{bmatrix} \cos\omega_1 t \\ \sin\omega_1 t \end{bmatrix} = \sqrt{\frac{2}{3}}\,I\,[1 + \cos 2(\omega_1 t + 30^0)]\begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - 120^0) \end{bmatrix}. \tag{21}$$

The instantaneous reactive power, q, of the load considered in this Illustration is not equal to zero, hence, according to the IRP p-q Theory, a reactive current has to occur in the supply lines. Its value in the α and β coordinates is equal to

$$i_{\alpha\mathrm{q}} = \frac{-u_\beta}{u_\alpha^2 + u_\beta^2}\,q = I \sin 2(\omega_1 t + 30^0)\sin\omega_1 t\,, \tag{22}$$

$$i_{\beta\mathrm{q}} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\,q = -I \sin 2(\omega_1 t + 30^0)\cos\omega_1 t\,, \tag{23}$$

and consequently, the inverse Clarke Transform results in the instantaneous reactive current in the supply lines

$$\begin{bmatrix} i_{\mathrm{Rq}} \\ i_{\mathrm{Sq}} \end{bmatrix} = I \sin 2(\omega_1 t + 30^0)\,\mathbf{C}^{-1}\begin{bmatrix} \sin\omega_1 t \\ -\cos\omega_1 t \end{bmatrix} = \sqrt{\frac{2}{3}}\,I \sin 2(\omega_1 t + 30^0)\begin{bmatrix} \sin\omega_1 t \\ \sin(\omega_1 t - 120^0) \end{bmatrix}. \tag{24}$$

Formulae (21) and (24) show that the names *instantaneous active current* and *instantaneous reactive current* were given in the IRP p-q Theory to currents that have nothing in common with the notion of the active and reactive currents as used in electrical engineering. Also the reactive current i_q occurs in supply lines of the load in spite of its zero reactive power, Q. Moreover, both the active and reactive currents in systems with sinusoidal voltage and linear loads that do not generate harmonics, as considered in the Illustration, are nonsinusoidal. For example, the active current in the line R is equal to

$$i_{\mathrm{Rp}} = \frac{I}{\sqrt{6}}[2\cos\omega_1 t + \cos(\omega_1 t + 60^0) + \cos(3\omega_1 t + 60^0)]\,, \tag{25}$$

thus, its contains the third order harmonic. This conclusion obtained from the IRP p-q Theory is in sharp contradiction to the notion of the active current that was introduced to electrical engineering by Fryze. The suggestion of the IRP p-q Theory that line currents of a linear load, that does not generate harmonics in a system with a sinusoidal voltage, contain a nonsinusoidal component should be considered as a major misconception of electrical phenomena in three-phase circuits. Moreover, unlike the active current, i_a, defined by Fryze, the active current, i_p, that results from the IRP p-q Theory is not the current that should remain in the supply lines after the load is compensated to unity power factor. It cannot be considered the compensation goal.

Illustration 1 also shows that the Instantaneous Reactive Power p-q Theory does not identify power properties of the load instantaneously. Both powers are time-varying quantities, so that, a pair of their values at any single instant of time does not identify power properties of the load. The possibility of instantaneous identification of the active and reactive powers, p and q, does not mean that power properties of the load are identified instantaneously. For example, for $t = \tau$, such that $(\omega_1 \tau + 30^0) = 0$, the instantaneous reactive power, $q = 0$, as it is in a circuit with purely resistive balanced load, while for $t = \tau$, such that $(\omega_1 \tau + 30^0) = 90^0$, both the instantaneous active and reactive powers are equal to zero. Thus, power properties of the load cannot be identified without observation of the p and q powers over the entire cycle of their variability. However, even such an observation of these powers does not explain power properties of the load without additional analysis. For example, the instantaneous reactive power, q, has occurred in the circuit considered in Illustration 1 not because of the load reactive power, Q, but because of the load imbalance. How could this imbalance be identified having the values of the instantaneous reactive power q? Therefore, the IRP p-q Theory has no advantages with respect to the time interval needed for the identification of load properties over power theories based on the frequency-domain approach that require the system to be observed over one period, T, of its variability.

Illustration 1 has demonstrated that the instantaneous reactive current has nothing in common with the load reactive power, Q. It also occurs that the instantaneous active current in the IRP p-q Theory has nothing in common with the load active power, P. This is shown in Illustration 2.

Illustration 2. Let us consider a circuit with a purely reactive load as shown in Fig. 3, supplied like in Illustration 1, thus, with the voltage in the α and β coordinates specified by formula (15).

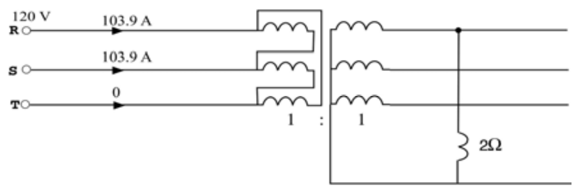

Fig. 3.

The line currents in such a circuit are equal to

$$i_{\mathrm{R}} = \sqrt{2}\, I \cos(\omega_1 t - 60^0), \qquad i_{\mathrm{S}} = -\, i_{\mathrm{R}}, \qquad i_{\mathrm{T}} = 0, \qquad I = 103.92 \text{ A}, \tag{26}$$

thus, the line currents in the α and β coordinates are equal to

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}\, I\cos(\omega_1 t - 60^{\mathrm{o}}) \\ -\sqrt{2}\, I\cos(\omega_1 t - 60^{\mathrm{o}}) \end{bmatrix} = \begin{bmatrix} \sqrt{3}\, I\cos(\omega_1 t - 60^{\mathrm{o}}) \\ -I\cos(\omega_1 t - 60^{\mathrm{o}}) \end{bmatrix}. \tag{27}$$

Consequently, the instantaneous real and imaginary powers have the values

$$p = u_\alpha i_\alpha + u_\beta i_\beta = \sqrt{3}\, U I \cos(2\omega_1 t - 30^0). \tag{28}$$

$$q = u_\alpha i_\beta - u_\beta i_\alpha = -\sqrt{3}\, U I\, [1 + \sin(2\omega_1 t - 30^0)]. \tag{29}$$

In spite of the zero active power of the load, there is a non-zero active current in the circuit. Its value in the α and β coordinates is equal to

$$i_{\alpha\mathrm{p}} = \frac{u_\alpha}{u_\alpha^2 + u_\beta^2}\, p = \frac{I}{2}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)], \tag{30}$$

$$i_{\beta\mathrm{p}} = \frac{u_\beta}{u_\alpha^2 + u_\beta^2}\, p = -\frac{I}{2}[\sin(\omega_1 t - 30^0) - \sin(3\omega_1 t - 30^0)], \tag{31}$$

and in the phase coordinates is equal to

$$\begin{bmatrix} i_{\mathrm{Rp}} \\ i_{\mathrm{Sp}} \end{bmatrix} = \mathbf{C}^{-1} \begin{bmatrix} i_{\alpha\mathrm{p}} \\ i_{\beta\mathrm{p}} \end{bmatrix} = \begin{bmatrix} \sqrt{2/3}, & 0 \\ -1/\sqrt{6}, & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{I}{2}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)] \\ -\frac{I}{2}[\sin(\omega_1 t - 30^0) - \sin(3\omega_1 t - 30^0)] \end{bmatrix}. \tag{32}$$

In particular, the instantaneous active current in the line R is equal to

$$i_{\mathrm{Rp}} = \frac{I}{\sqrt{6}}[\cos(\omega_1 t - 30^0) + \cos(3\omega_1 t - 30^0)]. \tag{33}$$

This means that according to the IRP p-q Theory an active current occurs even in purely reactive circuits. This current is nonsinusoidal even if there is no source of harmonics in the supply source and the load.

Illustrations 1 and 2 show some features of the IRP p-q Theory when it is applied to systems with sinusoidal voltages and currents without any explanation of these features. They can be explained using the Theory of the Currents' Physical Components.

3. Apparent Power Related Ambiguity

Power theory provides definitions of various powers in electrical circuits along with relations between them. Unfortunately, there is a major ambiguity with respect to one of the most commonly used powers, namely the apparent power, S, in three-phase systems. This ambiguity exists even at sinusoidal voltages and currents [3]. Namely, according to the IEEE Standard Dictionary of Electrical and Electronics Terms [19], the apparent power is defined as

$$S_A = U_R I_R + U_S I_S + U_T I_T \,, \tag{34}$$

and this power is referred to as arithmetical apparent power or as

$$S_G = \sqrt{P^2 + Q^2} \,, \tag{35}$$

referred to as geometrical apparent power. There exists a third definition of the apparent power for three-phase three-wire systems introduced for such systems under nonsinusoidal conditions in Ref. [17], but not referenced by the IEEE Standard Dictionary, namely

$$S = ||\boldsymbol{u}|| \cdot ||\boldsymbol{i}|| \,. \tag{36}$$

For systems with sinusoidal voltages and currents this apparent power is equal to

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2} \;\cdot \sqrt{I_R^2 + I_S^2 + I_T^2} \,, \tag{37}$$

and was suggested [20] by Buchholz in 1922. In balanced three-phase systems with sinusoidal voltages and currents these three apparent powers have the same numerical value. The difference becomes visible when the load is unbalanced or waveforms are nonsinusoidal. This difference in a circuit with an unbalanced load is shown in Illustration 3.

Illustration 3. For the system shown in Figure 2, definitions (34), (35) and (37) result in

$$S_A = 24.9 \text{ kVA}, \quad S_G = 21.6 \text{ kVA}, \quad S_B = 30.5 \text{ kVA},$$

and consequently, the power factor, $\lambda = P/S$, that means the ratio of the active and apparent power depends on the selection of the apparent power definition. Since the active power of the load in the Illustration considered is equal to P = 21.6 kW, then, depending on the selection of the apparent power definition, different values of the power factor are obtained, namely:

$$\lambda_A = 0.87, \qquad \lambda_G = 1, \qquad \lambda_B = 0.71.$$

Thus, it seems to be unclear what is the true value of the power factor. It is unclear as well, what is the power rating of a compensator, S_c, needed for the power factor improvement to unity value, that means

$$S_c = \sqrt{S^2 - P^2} \,. \tag{38}$$

The result depends on the selection of the apparent power definition. Unfortunately, any relation between powers in three-phase circuits cannot be established without due clarification of this ambiguity related to the apparent power definition.

4. Selection of the Apparent Power Definition

The apparent power is not associated with a particular power phenomenon in electrical circuits. It is a conventional quantity, a figure of merit that describes the supply equipment with respect to the voltage and current RMS values the supply is capable to provide customers. The same is with the power factor, λ. It is the ratio of the active and apparent powers, P/S. Its decline at a specified load active power, P, is associated with an increase in the RMS value of the supply current and consequently, with an increase of the active power loss in the supply. Therefore, the power loss in the supply can provide a key for the selection [3] of the true value of the power factor and consequently, for the selection of the apparent power definition. This selection can be based on the reasoning presented in the following Illustration.

Illustration 4. Resistances in the balanced system shown in Figure 4 were selected in such a way that, at the distribution voltage U = 220 V, the load active power P = 100 kW while the active power loss in the supply source amounts to 5 percent of the load active power, that means, ΔP_s = 5 kW. Because the load is balanced, all three definitions of the apparent power result in the same value of $S_A = S_G = S_B$ = 100 kVA.

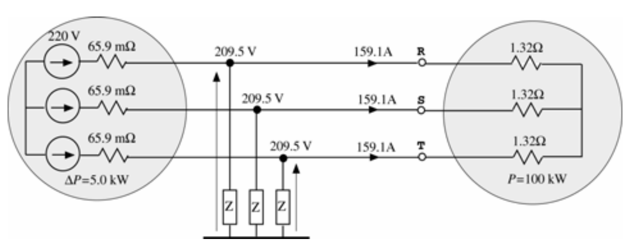

Fig. 4.

In the next step of this reasoning, the resistance of the unbalanced load, shown in Fig. 5, was calculated, at the assumption that this load, supplied from the same source has the same active power P = 100 kW. Since the load is unbalanced, definitions (34), (35) and (37) result in different values of apparent power, namely

$$S_A = 119 \text{ kVA}, \quad S_G = 100 \text{ kVA}, \quad S_B = 149 \text{ kVA},$$

and in different values of the power factor

$$\lambda_A = 0.84, \quad \lambda_G = 1, \quad \lambda_B = 0.67.$$

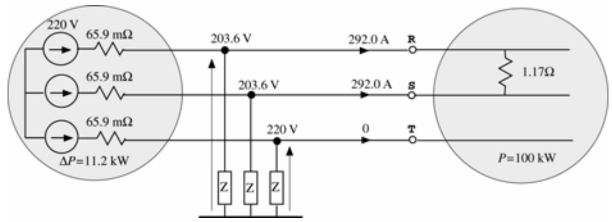

Fig. 5.

Observe, that when energy is delivered to the unbalanced load the active power loss in the supply increases from 5 to 11.2 kW, thus the purely resistive load in the circuit shown in Fig. 5 cannot be considered as load with unity power factor. Calculation of the power factor using the geometrical apparent power results in just such a unity power factor. This disqualifies the geometrical apparent power, S_G, as an acceptable definition of the apparent power in the presence of load imbalance. However, the question: *which value λ_A or λ_B represents the true value of the power factor?* still remains unanswered.

Since the value of apparent power of balanced loads is independent of this power definition, we could ask the question: *what is the apparent power of a balanced load with the active power P = 100 kW, that causes the same active power loss in the supply, that is ΔP_s = 11.2 kW, as the unbalanced load shown in Fig. 5*? A balanced RL load has to be found that satisfies the requirements as to powers P and ΔP_s values to answer such a question. Solution of such a problem results in the circuit shown in Fig. 6. For such a circuit

$$S_A = S_G = S_B = 149 \text{ kVA},$$

and its the power factor

$$\lambda_A = \lambda_G = \lambda_B = 0.67.$$

It means that the unbalanced load as shown in the Fig. 5 is equivalent with respect to the power loss in the supply to the balanced RL load shown in Fig. 6 that has power factor $\lambda = 0.67$.

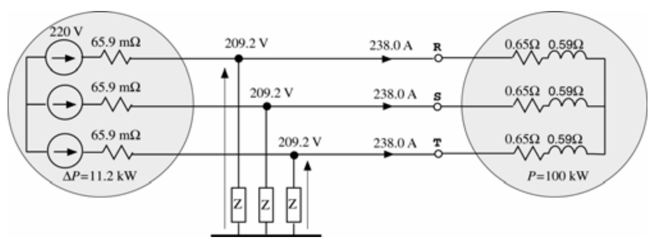

Fig. 6.

Thus, the power factor decline due to the load imbalance is associated with a true value of the power loss increase if the apparent power S is calculated according to the definition

$$S = ||\boldsymbol{u}|| \cdot ||\boldsymbol{i}|| = \sqrt{U_R^2 + U_S^2 + U_T^2} \cdot \sqrt{I_R^2 + I_S^2 + I_T^2} \,. \tag{39}$$

The arithmetic and geometric definitions of the apparent powers as suggested by the IEEE Standard Dictionary [19] do not characterize true loading of the supply source with respect to active power loss ΔP_s in the supply and the true power factor. Therefore, the apparent power, S, in the Theory of the Currents' Physical Components is defined according to definition (36).

5. The CPC Theory of Three-Phase Systems under Sinusoidal Conditions

The Theory of the Currents' Physical Components was developed in Ref [17] for three-phase, three-wire systems with nonsinusoidal voltages and currents. It is used in this paper for analysis of the IRP p-q Theory applied to three-phase, three-wire systems with sinusoidal voltages and current. Therefore, only a reduced version of CPC Theory is needed, as drafted in this section.

Let a linear load as shown in Fig. 7 is supplied with a symmetrical voltage of the positive sequence, such that $u_R(t) = \sqrt{2}\, U_R \cos\omega_1 t$. The load can be characterized by two admittances. Namely, by the equivalent admittance

$$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR} \,, \tag{40}$$

and the unbalanced admittance

$$\boldsymbol{A} = Ae^{j\psi} = -(\boldsymbol{Y}_{ST} + \alpha \boldsymbol{Y}_{TR} + \alpha^{*}\boldsymbol{Y}_{RS}), \qquad \alpha = 1 e^{j120^{0}}. \tag{41}$$

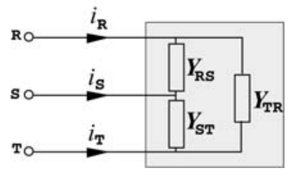

Fig. 7

If the complex RMS values of the line-to-artificial zero point voltages of the supply are arranged into vectors

$$\begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_T \end{bmatrix} = \boldsymbol{U}, \qquad \begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_T \\ \boldsymbol{U}_S \end{bmatrix} = \boldsymbol{U}^{\#}, \tag{42}$$

then, the supply current of the load

$$\boldsymbol{i} = \begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} e^{j\omega_1 t} = \sqrt{2}\,\mathrm{Re}\,\boldsymbol{I}\, e^{j\omega_1 t}, \tag{43}$$

can be decomposed into three components, namely

$$\boldsymbol{i} = \boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u, \tag{44}$$

where

$$\boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re}\{G_e\, \boldsymbol{U}\, e^{j\omega_1 t}\}, \tag{45}$$

is the active current,

$$\boldsymbol{i}_r = \sqrt{2}\,\mathrm{Re}\{jB_e\, \boldsymbol{U}\, e^{j\omega_1 t}\}, \tag{46}$$

is the reactive current and

$$\boldsymbol{i}_u = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{A}\, \boldsymbol{U}^{\#} e^{j\omega_1 t}\}, \tag{47}$$

is the unbalanced current.

These three currents are mutually orthogonal, that means their scalar product, defined for periodic, three -phase quantities $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ with the period T as

$$(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^{T}(t)\, \boldsymbol{y}(t)\, dt, \tag{48}$$

is equal to zero. Therefore, the RMS values of these currents satisfy the relationship

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2, \tag{49}$$

where

$$\|\boldsymbol{i}_a\| = G_e\, \|\boldsymbol{u}\|, \quad \|\boldsymbol{i}_r\| = |B_e|\, \|\boldsymbol{u}\|, \quad \|\boldsymbol{i}_u\| = A\, \|\boldsymbol{u}\|. \tag{50}$$

Multiplying equation (49) by the square of the supply voltage RMS value, $\|\boldsymbol{u}\|^2$, the power equation of three-phase loads with sinusoidal voltages and currents is obtained, namely

$$S^2 = P^2 + Q^2 + D^2, \tag{51}$$

where

$$D = \|\boldsymbol{u}\|\, \|\boldsymbol{i}_u\| = A\, \|\boldsymbol{u}\|^2, \tag{52}$$

is the unbalanced power of the load, while

$$P = \|\boldsymbol{u}\|\, \|\boldsymbol{i}_a\| = G_e\, \|\boldsymbol{u}\|^2, \qquad Q = \pm\|\boldsymbol{u}\|\, \|\boldsymbol{i}_r\| = -B_e\, \|\boldsymbol{u}\|^2. \tag{53}$$

Equation (51), along with definitions (52) and (53), emphasize the dependence of the load apparent power S on three independent power phenomena of three-phase systems with sinusoidal voltages and currents. (i) Permanent energy transmission and associated active power, P, (ii) the current phase-shift due to the presence of reactive elements in the load and associated reactive power, Q, and (iii) the load imbalance related supply current asymmetry and associated unbalanced power, D. Because the active, reactive and unbalanced currents, $\boldsymbol{i}_a$, $\boldsymbol{i}_r$ and $\boldsymbol{i}_u$, are associated with distinctive physical phenomena in the circuit, these currents are referred to as *physical components of the current*.

Let us apply the CPC Theory to the circuit considered in Illustration 1. Since the line-to-line admittances of the load $\boldsymbol{Y}_{RS} = 0.5$ S, $\boldsymbol{Y}_{ST} = \boldsymbol{Y}_{TR} = 0$, the formula (40) results in the equivalent admittance,

$$\boldsymbol{Y}_e = G_e + jB_e = \boldsymbol{Y}_{RS} + \boldsymbol{Y}_{ST} + \boldsymbol{Y}_{TR} = 0.5 \text{ S}.$$

and the formula (41) results in the unbalanced admittance

$$\boldsymbol{A} = -(\boldsymbol{Y}_{ST} + \alpha \boldsymbol{Y}_{TR} + \alpha^* \boldsymbol{Y}_{RS}) = -\alpha^* \boldsymbol{Y}_{RS} = 0.5 e^{j60^0} \text{ S},$$

Consequently, the active current of the load is equal to

$$\boldsymbol{i}_a = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \sqrt{2}\,\text{Re}\{G_e \begin{bmatrix} U \\ U e^{-j120^0} \\ U e^{+j120^0} \end{bmatrix} e^{j\omega_1 t}\} = 60\sqrt{2} \begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - 120^0) \\ \cos(\omega_1 t + 120^0) \end{bmatrix} \text{ A}.$$

Since the equivalent susceptance, B_e, of the load considered is equal to zero, the reactive current does not occur in the supply current. The unbalanced current is equal to

$$\boldsymbol{i}_u = \begin{bmatrix} i_{Ru} \\ i_{Su} \\ i_{Tu} \end{bmatrix} = \sqrt{2}\,\text{Re}\{\boldsymbol{A} \begin{bmatrix} U \\ U e^{+j120^0} \\ U e^{-j120^0} \end{bmatrix} e^{j\omega_1 t}\} = 60\sqrt{2} \begin{bmatrix} \cos(\omega_1 t + 60^0) \\ \cos(\omega_1 t + 180^0) \\ \cos(\omega_1 t - 60^0) \end{bmatrix} \text{ A}.$$

Indeed, the sum of the active and unbalanced currents is equal to the line current of the load, namely

$$\boldsymbol{i}_a + \boldsymbol{i}_u = 103.9\sqrt{2} \begin{bmatrix} \cos(\omega_1 t + 30^0) \\ \cos(\omega_1 t - 150^0) \\ 0 \end{bmatrix} \text{ A} = \boldsymbol{i},$$

thus, the supply current cannot contain any component other than the active and unbalanced currents, which is consistent with the load properties. It is a resistive unbalanced load. Unlike in the case of the IRP p-q Theory, the same consistence is obtained when the CPC Theory is applied to the circuit shown in Figure 3. The supply current in this circuit is composed of only the reactive and unbalanced currents.

Having drafted the CPC Theory as it applies to systems with sinusoidal voltages and currents, it can be used now for analysis of some features of the IRP p-q Theory. A main concern is the relationship of the p and q powers to the load parameters and its active, reactive and unbalanced powers, P, Q and D.

6. Relationship between *p, q* and *P, Q, D* Powers

According to the CPC Theory, the supply current in line R can be expressed as

$$i_R = i_{Ra} + i_{Rr} + i_{Ru} = \sqrt{2}\,\text{Re}\{(G_e + jB_e + \boldsymbol{A})\,\boldsymbol{U}_R\, e^{j\omega_1 t}\}, \tag{54}$$

and the supply current in line S as

$$i_S = i_{Sa} + i_{Sr} + i_{Su} = \sqrt{2}\,\text{Re}\{(G_e + jB_e + \alpha^* \boldsymbol{A})\,\boldsymbol{U}_S\, e^{j\omega_1 t}\}. \tag{55}$$

The Clarke Transform (9) of the line currents results in

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \mathbf{C} \begin{bmatrix} i_R \\ i_S \end{bmatrix} = \mathbf{C} \begin{bmatrix} \sqrt{2}\,\text{Re}\{(G_e + jB_e + \boldsymbol{A})\,\boldsymbol{U}_R\, e^{j\omega_1 t}\} \\ \sqrt{2}\,\text{Re}\{(G_e + jB_e + \alpha^* \boldsymbol{A})\,\boldsymbol{U}_S\, e^{j\omega_1 t}\} \end{bmatrix} =$$

$$= \begin{bmatrix} \sqrt{3}\,\text{Re}\{(G_e + jB_e + \boldsymbol{A})\,\boldsymbol{U}_R\, e^{j\omega_1 t}\} \\ \sqrt{3}\,\text{Re}\{(-jG_e + B_e + j\boldsymbol{A})\,\boldsymbol{U}_R\, e^{j\omega_1 t}\} \end{bmatrix} =$$

$$= \sqrt{3}\,U_R \begin{bmatrix} G_e \cos\omega_1 t - B_e \sin\omega_1 t + A\cos(\omega_1 t + \psi) \\ G_e \sin\omega_1 t + B_e \cos\omega_1 t - A\sin(\omega_1 t + \psi) \end{bmatrix} \tag{56}$$

Taking into account that the supply voltage in the α and β coordinates is specified by formula (15) and having the line currents expressed in these coordinates, the instantaneous active power can be expressed in terms of the load equivalent parameters as follow

$$p = u_\alpha i_\alpha + u_\beta i_\beta = 3U_R^2[G_e + A\cos(2\omega_1 t + \psi)]. \tag{57}$$

Thus, the instantaneous active power is associated with two different phenomena, namely, with permanent energy conversion in the load, characterized by its equivalent conductance, G_e, and with the load imbalance, characterized by its unbalanced admittance, $\boldsymbol{A}$.

The instantaneous reactive power, expressed in terms of the load equivalent parameters, is equal to

$$q = u_\alpha i_\beta - u_\beta i_\alpha = 3U_R^2[B_e - A\sin(2\omega_1 t + \psi)] \; . \tag{58}$$

Thus, it is also associated with two different phenomena, with the presence of reactive elements in the load, characterized by its equivalent susceptance, B_e, and like the instantaneous active power, p, with the load imbalance. Taking into account that particular powers of the load in the Theory of the CPC be expressed in terms of the load equivalent parameters, thus

$$P = 3G_e U_R^2 \, , \quad Q = -\,3B_e U_R^2 \, , \quad D = 3A\,U_R^2 \, , \tag{59}$$

the instantaneous active and reactive powers in the IRP p-q Theory can be expressed as

$$p = P + D\cos(2\omega_1 t + \psi)] \, , \tag{60}$$

$$q = -Q - D\sin(2\omega_1 t + \psi)] \, . \tag{61}$$

These two relations between powers in the IRP p-q Theory and the active, reactive and unbalanced powers, P, Q and D explain the reasons for which this theory fails to identify power properties of three-phase loads. Each of these two powers is affected by two different power phenomena, thus they are internally involved quantities. Moreover, the load imbalance causes their time variability and consequently, power properties cannot be identified instantaneously. Further signal processing of the p and q powers is needed for identification of the load parameters, G_e, B_e and $\boldsymbol{A}$**,** as well as P, Q and D powers.

7. Power Factor Improvement and Compensator Energy Storage

Although the IRP p-q Theory is commonly used as a theoretical fundamental for the control algorithm of compensators for the power factor improvement, this theory does not provide an explicit relationship between p and q powers (or between active and reactive currents) and the power factor, λ. Although the IRP p-q Theory is applied mainly for compensator control in the presence of harmonics, some deficiencies of this theory in compensation related issues seem to be visible even in systems with sinusoidal voltages and currents. Namely, the power factor is the ratio of the active and apparent powers, $\lambda = P/S$, but these two powers are not directly identified by the IRP p-q Theory. These powers can be calculated after the entire period of the p and q powers variability is observed. The time needed for the power factor evaluation is not shorter than that when the formula

$$\lambda = \frac{||\boldsymbol{i}_a||}{||\boldsymbol{i}||} \, . \tag{62}$$

is used. Unlike the IRP p-q Theory, the CPC Theory provides, however, explicit criteria that have to be met by a compensator to improve the power factor to unity. Since formula (62) can be expressed as

$$\lambda = \frac{||\boldsymbol{i}_a||}{\sqrt{||\boldsymbol{i}_a||^2 + ||\boldsymbol{i}_r||^2 + ||\boldsymbol{i}_u||^2}} \, , \tag{63}$$

the supply source operates with unity power factor only if the reactive and unbalanced currents are totally compensated. Details of such compensation based on the CPC Theory are not the subject of this paper, however. Only some general issues related to energy oscillation and required energy storage by the compensator are discussed here.

The IRP p-q Theory handles the energy oscillation and required capability of the compensator for energy storage in terms of the mean and the oscillating component value of the p and q powers. Since the association of these powers with specific power phenomena in the circuit is ambiguous, also the issue of the relationship between the required capability of energy storage of the compensator and power properties of the load remains unclear.

When the CPC Theory is applied for that purpose, then the rate of energy flow from the supply source to a three-phase three-wire load can be expressed in terms of physical components of the current as a sum of instantaneous powers associated with the active, reactive and unbalanced currents, namely

$$\frac{dW}{dt} = p = \boldsymbol{u}^T(\boldsymbol{i}_a + \boldsymbol{i}_r + \boldsymbol{i}_u) = p_a + p_r + p_u \, . \tag{64}$$

The term $p_a = \boldsymbol{u}^T\boldsymbol{i}_a$, represents the component of the instantaneous power, p, that occurs when the load has the active power, P. Therefore, only this part of the instantaneous power p should be referred to as the *instantaneous active power*. The term $p_r = \boldsymbol{u}^T\boldsymbol{i}_r$, represents the component of the instantaneous power, p, that occurs when the load has the reactive power, Q. Therefore, only this part of the instantaneous power p should be referred to as the *instantaneous* *reactive power*. The term $p_u = \boldsymbol{u}^T\boldsymbol{i}_u$, represents the component of the instantaneous power, *p*, that occurs when the load has the unbalanced power. Taking into account that

$$\boldsymbol{u} = \begin{bmatrix} u_R \\ u_S \\ u_T \end{bmatrix} = \sqrt{2}\,U_R \begin{bmatrix} \cos\omega_1 t \\ \cos(\omega_1 t - 120^0) \\ \cos(\omega_1 t + 120^0) \end{bmatrix}, \tag{65}$$

and the physical components of the current are defined by formulae (45)-(47), particular instantaneous powers are equal to

$$p_a = \boldsymbol{u}^T\boldsymbol{i}_a \equiv P\,, \tag{66}$$

$$p_r = \boldsymbol{u}^T\boldsymbol{i}_r \equiv 0\,, \tag{67}$$

$$p_u = \boldsymbol{u}^T\boldsymbol{i}_u = D\cos(2\omega_1 t + \psi)\,. \tag{68}$$

Thus, energy oscillation between the load and the supply source is associated only with the presence of unbalanced current.

A compensator that improves the power factor to unity value must compensate the reactive and unbalanced currents. Let us suppose, that these currents are compensated by two separate compensators as shown in Fig. 8, namely, a compensator of the reactive current that has the current $\boldsymbol{i} = -\boldsymbol{i}_r$ and a compensator of the unbalanced current that has current $\boldsymbol{i} = -\boldsymbol{i}_u$.

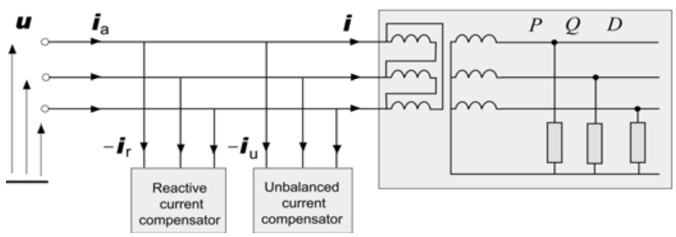

Fig. 8.

Since at each instant of time, $p_r = 0$, there is no energy flow between the supply source and the compensator of the reactive current i_r, therefore, energy storage is not needed for the reactive current compensation. There is however, energy oscillation, with the power p_u specified by formula (68), between the compensator of the unbalanced current i_u and the supply source. Such a compensator of unbalanced current cannot be built without a sufficient capability of energy storage. Since the frequency of energy oscillation is equal to $2\omega_1$, the compensator has to be able to store, at least, energy

$$W_s = \int_0^{T/4} p_u\, dt = D \int_0^{T/4} \sin 2\omega_1 t\, dt = \frac{T}{2\pi} D\,. \tag{69}$$

The compensator for the power factor improvement is built, of course, as a single device that compensates both reactive and unbalanced current. Formula (69) specifies the required energy storage capability necessary for such a compensator.

8. Conclusions

The analysis presented in this paper demonstrates that the IRP p-q Theory considered as a power theory, i.e., a theory that explains and describes power related phenomena in electrical circuits, has major deficiencies. It does not identify power properties of three-phase systems even under sinusoidal conditions. It identifies these properties only under the conditions that the load is balanced. A three-phase, three-wire balanced system under sinusoidal conditions is a trivial case, however. That case can be reduced for analysis to a single-phase system. Nonetheless, it is a clear advantage of the IRP p-q Theory over other ones that in balanced sinusoidal systems, where $P = q$ and $Q = -p$, it enables to calculate the active and reactive powers instantaneously. Unfortunately, when the load is unbalanced then the *p* and *q* powers are time variant quantities that do not provide instantaneous information on power properties of the load. Indeed, both powers are involved quantities, dependent at the same time on two different power phenomena. An additional analysis is required to identify the active, reactive, unbalanced and apparent powers after the *p* and *q* powers are recorded over the entire period of their variability. Consequently, the IRP p-q Theory, although it is based on a time-domain approach to power theory, has no advantages over the frequency-domain approach with respect to the time interval needed for identification of power properties of three-phase loads.

According to the IRP p-q Theory, the instantaneous reactive current can occur in linear circuits without reactive elements and consequently, when the reactive power is zero. Moreover, the instantaneous active current can occur when the active power is zero. These conclusions from the IRP p-q Theory contradict the common meanings of the active and reactive currents. Moreover, these two currents in systems with sinusoidal supply voltage and without any loads that generate harmonics are nonsinusoidal. It means, the IRP p-q Theory misinterprets power phenomena in electrical circuits. Moreover, it does not reveal the load imbalance as the cause of power factor degradation. It interprets the load imbalance as a loading that causes only a change of the active and reactive currents.

The analysis presented in this paper does not evaluate the usefulness of the Instantaneous Reactive Power p-q Theory for developing control algorithms of active power filters. Among other reasons, also because such filters are built to compensate the load generated current harmonic, while analysis in this paper was confined to systems under sinusoidal conditions. Nonetheless, the results of this analysis raise a question: *is the instantaneous compensation possible if power properties of three-phase loads cannot be identyfied instantaneously?* Expectations towards power theory are, of course, different than towards control algorithm of a compensator. However, one could ask a question: *is the control credible if it is based on a power theory that misinterprets power phenomena?*

9. References

[1] H. Akagi, Y. Kanazawa and A. Nabae, "Instantaneous reactive power compensators comprising switching devices without energy storage components," *IEEE Trans. on IA,* IA-20, 1984, No. 3, pp. 625-630.

[2] H. Akagi and A. Nabae, "The p-q theory in three-phase systems under non-sinusoidal conditions," *Europ. Trans. on Electrical Power, ETEP,* Vol. 3, No. 1, January/February 1993, pp. 27-31.

[3] L.S. Czarnecki, "Energy flow and power phenomena in electrical circuits: illusions and reality," *Archiv fur Elektrotechnik,* (82), No. 4, 1999, pp. 10-15.

[4] M. Depenbrock, D.A. Marshal and J.D. van Wyk, "Formulating requirements for universally applicable power theory as control algorithm in power compensators " *Europ. Trans. on Electrical Power, ETEP,* Vol. 4, No. 6, Nov./Dec. 1994, pp. 445-456.

[5] W. le Roux and J.D. van Wyk, "Evaluation of residual network distortion during compensation according to the 'Instantaneous Power Theory," *European Trans. on Electrical Power, ETEP*, Vol. 8, No. 5, pp. 337-344, Sept./Oct. 1998.

[6] P. Rossetto and P. Tenti, "Evaluation of instantaneous power terms in multi-phase systems: techniques and application to power-conditioning equipment," *Europ. Trans. on Electrical Power, ETEP,* Vol. 4, No. 6, Nov./Dec. 1994, pp. 469-475.

[7] F.Z. Peng, G.W. Ott, Jr. and D.J. Adams, "Harmonic and reactive power compensation based on the generalized instantaneous reactive power theory for three-phase four-wire systems," *IEEE Trans. on Power Electronics,* Vol. 13, No. 6, Nov. 1998, pp. 1174-1181.

[8] S-J. Huang and J-C Wu, "A control algorithm for three-phase three-wired active power filters under nonideal mains voltages," *IEEE Trans. on Power Electronics,* Vol. 14, No. 4, July 1999, pp. 753-760.

[9] K. Kim, F. Blaabjerg, B. Bak Jensen and J. Choi, "Instantaneous power compensation in three-phase system using p-q-r theory", *IEEE Trans. on Power Electr.*, Vol. 17, No. 5 Sept. 2002, pp. 701-710.

[10] J.L. Willems, "A new interpretation of the Akagi-Nabae power components for nonsinusoidal three-phase situations," *IEEE Trans. on Instr. Measur.,* Vol. 41, No. 4, pp 523-527, August 1992.

[11] F.Z. Peng and J-S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," *IEEE Trans. Instr. Measur.,* Vol. IM-45, No. 1, pp. 293-297, Feb. 1996.

[12] L. Cristaldi and A. Ferrero, "Mathematical foundations of the instantaneous power concept: an algbraic approach," *European Trans. on Electrical Power, ETEP,* Vol. 6, No. 5, pp. 305-309, Sept./Oct. 1996.

[13] E.H. Watanabe, M. Stephan and M. Aredes, "New concept of instantaneous active and reactive powers in electrical systems with generic loads," *IEEE Trans. on Power Delivery,* Vol. 8, No. 2, April, 1993, pp. 697-703.

[14] P. Salmeron and J.C. Montano, "Instantaneous power components in polyphase systems under nonsinusoidal conditions," *IEE Proc. on Science, Measurement and Technology*, Vol. 143, No. 2, Feb.1996, pp. 239-297.

[15] K. Kim, F. Blaabjerg, B. Bak Jensen, "Spectral analysis of instantaneous powers in single-phase and three-phase systems with use of p-q-r theory", *IEEE Trans. on Power Electronics*, Vol. 17, No. 5 Sept. 2002, pp. 711-720.

[16] J.M. Morendaz-Eguilaz and J. Peracaula, "Understanding AC power using the generalized instantaneous reactive power theory: considerations for instrumentation of three-phase electronic converters," *Proc. of the IEEE Int. Symp., Ind. Electr.,* ISIE '99, Vol. 3, pp. 1273-1277.

[17] L.S. Czarnecki, "Orthogonal decomposition of the current in a three-phase non-linear asymmetrical circuit with nonsinusoidal voltage," *IEEE Trans. IM.,* Vol. IM-37, No. 1, March 1988, pp. 30-34.

[18] S. Fryze, "Wirk-, Blind-, und Scheinleistung in Elektrischen Stromkreisen mit nicht-sinusoidalen Verlauf von Strom und Spannung," *ETZ,* No. 25, 26 and 27, 1932.

[19] *The New IEEE Standard Dictionary of Electrical and Electronics Terms*, Institute of Electrical and Electronics Engineers, Inc., New York, 1997.

[20] F. Buchholz, "Die Drehstrom-Scheinleistung ein ungleichmeβiger Belastung drei Zweige," *Licht und Kraft,* No. 2, January 1922, pp. 9-11.

[21] L.S. Czarnecki, “Harmonics and power phenomena,” *Wiley Encyclopedia of Electrical and Electronics Engineering*, John Wiley & Sons, Inc., Supplement 1, 2000, pp. 195-218.