Circuits with Semi-Periodic Currents: Main Features and Power Properties

L. S. Czarnecki

Abstract

The active, reactive, scattered and unbalanced currents and powers, the RMS value and harmonic components are defined only for periodic quantities. Unfortunately, non-periodic variation of load parameters or their periodic variation but with a frequency different than the frequency of the voltage produced by power generators, causes non-periodic currents. Such non-periodic currents, however, are often much lower than periodic components and consequently, the entire current can be considered as semi-periodic. The notion of semi-periodic voltage has similar meaning. This paper discusses the main properties of semi-periodic currents, it provides their classification and introduces a concept of primary, secondary, coperiodic, non-coperiodic components as well as the concept of interharmonic noise and quasi-harmonics. The paper investigates how circuits with semi-periodic voltages and currents could be described in power terms.

1 Introduction

Time-variance of load parameters may cause that load currents loose their periodicity. Before power electronics were developed and electric energy flow was controlled mainly with mechanical switches, only such switches, disturbances, such as lightnings or short-circuits and mechanical power variation were the main cause of loads parameter variability. Switching operations, arc furnaces and spot welders were the main sources of non-periodic currents and such currents occurred relatively seldom. Consequently, power phenomena in circuits with non-periodic currents were not a subject of investigation, especially when such phenomena even in circuits with periodic currents were not sufficiently well comprehended.

Power electronics facilitates fast switching operations for energy flow control and consequently, loads with time-varying parameters are much more common now than they were before. Also, much more common are non-periodic currents.

Like periodic distortion of voltages and currents, non-periodicity may contribute to a decline in the effectiveness of energy transmission. Also, some undesirable effects may occur. Light flickering is one of the best known of such harmful effects.

The harmful effects of non-periodic components of voltages and currents are in many respects similar to that caused by harmonics. However, they are even much more elusive. They may contribute to power loss, to disturbances, to measurement errors and control malfunctions, thus to degradation of the supply quality in distribution systems.

Some non-periodic phenomena such as voltage notches caused by short-circuits in the power system or lightning spikes that affect the supply quality have been a matter of concern and studies, due to the damages they can cause, for a long time. However, a systematic approach to the power factor improvement in circuits with non-periodic waveforms requires, similarly as in the case of circuits with harmonic distortions, that the power phenomena in the circuit are well understood. Unfortunately, it seems that the general knowledge regarding the effect of non-periodic components of voltages and currents on power phenomena in distribution systems is reminiscent now of the knowledge on effects of harmonics we had a few decades ago. There are more questions than answers now. Fortunately, power phenomena in circuits with periodic voltages and currents are comprehended at last and can serve as a starting point and a reference for studies on such phenomena in circuits with non-periodic waveforms.

The duration of a non-periodic component could be a tiny fraction of a 50 or 60 Hz frequency cycle, it could be comparable with a single or a few cycles or it could be considered as a permanent component of voltages or currents. Non-periodic phenomena may have repetitiveness with a fixed or with a variable frequency, or they could be random processes. A great variety of extremely dispersed features make the non-periodic phenomena in distribution systems difficult to handle as a whole. Because of these features, any mathematical tool, equally powerful as the Fourier series in linear systems with periodic voltages and currents, has not been developed for systems with non-periodic waveform yet. Spectral analysis, based on the Fourier Transform, still competes with time-frequency representations, such as wavelets, or with a statistical approach.

The class of non-periodic waveforms is much richer with respect to their features than the class of periodic waveforms. Consequently, taking into account how long the power phenomena in circuits with periodic waveforms were debated, it is unlikely that these phenomena in circuits with non-periodic waveforms will be identified in a short time. The main intention of the author of this paper is to initiate investigations on power phenomena in circuits with non-periodic volt-
ages and currents, as well as to suggest a classification of non-periodic waveforms in distribution systems and to define some basic terms.

2 Non-periodic quantities

The definition of a non-periodic quantity is trivial: any quantity that does not have a period is a non-periodic quantity. It means that, there is not such a non-zero number \( T \), such that the identity

\[
x(t) = x(t - nT)
\]

is fulfilled for all integers \( n \). As a consequence of the lack of periodicity, the basic functionals upon which the power theory of circuits with periodic quantities is built, namely, the norm

\[
\|x\| = \sqrt{\frac{1}{T} \int_0^T x^2(t) \, dt},
\]

and the scalar product of two non-periodic quantities, \( x_1(t) \) and \( x_2(t) \), defined as

\[
(x_1, x_2) = \frac{1}{T} \int_0^T x_1(t) x_2(t) \, dt,
\]

do not exist. Also, such a quantity cannot be expressed in the form of Fourier series

\[
x(t) = X_0 + \sqrt{2} \text{ Re} \sum_{n=1}^{\infty} X_n e^{j n \omega_0 t},
\]

since the integral

\[
X_n = \frac{\sqrt{2}}{T} \int_0^T x(t) e^{-j n \omega_0 t} \, dt,
\]

cannot be calculated. The lack of the norm and the scalar product causes that neither the RMS value nor the active power can be calculated in circuits with non-periodic voltages and currents. Consequently, no power quantity, apart from the instantaneous power, can be calculated or measured based on formulae used in circuits with periodic voltages and currents.

3 Primary and secondary components

Non-periodic voltages and currents in electrical power systems usually have some common particular properties. They occur as an effect of disturbance or conversion of periodic quantities that have a frequency determined by the power system generators, usually of 50 or 60 Hz, referred to as a “primary frequency” and denoted by \( f_p \) in this paper. The term “primary period”, \( T_p = 1/f_p \), will be used as well. Moreover, and this is crucial for the power phenomena in such systems, only at such a frequency can the energy be conveyed permanently from generators to loads. Consequently, the voltage and the current components of the frequency as produced by the power system generators have a particular importance. They will be called “primary” components in this paper and will be denoted by index “p” or by the index that specifies the frequency of such a component, for example, by the symbols \( x_0(t) \), \( x_50(t) \) or \( x_{60}(t) \). Usually, the primary component is sinusoidal and in such a case it could be referred to as a fundamental component or harmonic. When the generated voltage is non-sinusoidal, also the primary current is non-sinusoidal, therefore, the term “fundamental” is not applied to it in this paper. The supply current of linear, time-invariant loads can contain only a primary component.

Time-variance and non-linearity of load parameters may cause that the supply current contains a component different than the primary component. Such a component will be referred to as a “secondary” component in this paper. The secondary component can be periodic or non-periodic. In the case of only non-linear loads, the secondary component is periodic. The secondary component will be denoted generally by \( x_s(t) \) in this paper, or by \( i_s(t) \).

When the primary voltage is sinusoidal and the load, due to its non-linearity, generates harmonics, then the load current secondary component, \( i_s(t) \), contains all harmonics and can be referred to as a distorting current. The ferroresonance phenomenon caused by inductance non-linearity in a resonant circuit, can generate sub-harmonics, that means, current components of the frequency \( f_p/3, f_p/5, f_p/7, \ldots \). The load current remains periodic, however, only its period increases to \( 3 T_p, 5 T_p, 7 T_p, \ldots \) respectively.

4 Coperiodic and non-coperiodic components

The secondary component of the load current in circuits with time-invariant parameters always has a period that is common with the period of the primary component, and therefore, it will be referred to as a “coperiodic” component of the current. Because of the common period, the coperiodic component cannot cause non-periodicity of the load current. The load current in such a circuit is non-periodic only if the secondary component is non-periodic or if this component is periodic but has no common period with the primary component. Such a current component will be referred to [9] as a “non-coperiodic” component.

From the mathematical point of view, two periodic functions are non-coperiodic only if the ratio of their frequencies is not a rational number. Consequently, a secondary component with the frequency, for example, \( f_s = 51.5 \) Hz is coperiodic in a system with the primary frequency of \( f_p = 50 \) Hz, since the ratio 50/51.5 = 50/515 is a rational number. Their common period is equal to \( T = 500 T_p = 515 T_p = 10 \) s. Thus, when the frequency of the secondary component is rounded by a finite number of digits, then such a component is coperiodic in the mathematical sense. Its frequency in electrical circuits can be random however, or at least, there is no fixed relationship between frequencies of the primary and secondary components. Consequently, their common period could be random and this makes the current periodicity useless for energy flow analysis and measurement. Therefore, if there is not a fixed relationship
between frequencies of the secondary and primary components, then the secondary component is consid-
ered as non-coperiodic and such quantities are consid-
ered as non-periodic in this paper, even if they are periodic in the mathematical sense.

Illustration 1. The current ,

\[ i(t) = I_{ma} \sin (2\pi 50t) + I_{mb} e^{-j} \sin \left(\sqrt{2}(2\pi 50t)\right), \]

is a classic example of a non-periodic current composed of two periodic components. The second component is a non-coperiodic, however, with the first one.

Illustration 2. The current

\[ i(t) = I_{ma} \sin (2\pi 50t) + I_{mb} e^{-j} \sin \left(\sqrt{2}(2\pi 50t)\right), \]

is non-periodic with a non-periodic second component. The first component is coperiodic with the supply voltage of 50 Hz frequency.

5 Semi-periodic quantities

Because of the presence of the primary component, usually sinusoidal, non-periodic currents in electrical power systems can be considered as a sub-class of non-periodic quantities. They will be referred to as “semi-periodic” quantities in this paper. A semi-periodic current can be defined as a current response of a time-variant load for a periodic supply voltage. Having such a definition, the effect of non-periodic components in the supply voltage, such as lightning spikes or transient notches, on the load current are neglected. Indeed, they are important for the system reliability, but not for power phenomena.

The difference between periodic, non-periodic and semi-periodic quantities can be analyzed and discussed in terms of their Fourier Transform, namely

\[ X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt. \] 

The frequency spectrum, \( X(j\omega) \), is not bounded as to the form if quantity \( x(t) \) is non-periodic. Essentially, this spectrum is continuous although, if \( x(t) \) contains periodic components, it may also contain infinite spikes.

The spectrum \( X(j\omega) \) of periodic quantities is discrete, namely, if it has the Fourier series (4), then, assuming for simplicity sake, that \( X_0 = 0 \), the quantity \( x(t) \) has the spectrum \( [1] \)

\[ X(j\omega) = \sqrt{2\pi} \sum_{n=-\infty}^{\infty} X_n \delta(\omega - n\omega_1), \]

where the symbol \( \delta(\omega) \) denotes the Dirac delta impulse. It means, it has the form of a sequence of evenly distributed infinite impulses at frequencies of \( n\omega_1 \), where \( \omega_1 \) is the primary frequency. If the quantity \( x(t) \) does not have periodic components, its spectrum is a continu-
ous function. The spectrum of semi-periodic quantities is a combination of a continuous function and a sequence of Dirac’s delta impulses.

When the periodic and non-periodic components of the quantity \( x(t) \) are additive,

\[ x(t) = x_p(t) + x_n(t), \]

then its spectrum is a sum of the spectra of these com-
ponents and can be presented in the form shown in Fig. 1.

The part of the spectrum between harmonic frequencies shall be referred to as “interharmonic noise”. The term “interharmonics”, used in some publications [3-5] on non-periodic quantities, for oscillations between “harmonics”, will not be used in this paper.

These oscillations have essentially random frequency and could be consider as the interharmonic noise. Moreover, the frequency of an interharmonic component is much less crucial than a harmonic frequency. For example, the second order harmonic behaves in a three phase system quite differently than the third order har-
monic. Short-time waveform disturbances and/or non-
coperiodic components of a random frequency contrib-
ute mainly to the interharmonic noise. These distur-
bances and non-coperiodic components can be consid-
ered as additive with respect to the primary component, \( x_1(t) \).

Non-periodicity may occur as a result of slow variation of the load parameters. The semi-periodic quanti-
ty can be expressed as a product of the primary quantity and modulating function, namely

\[ x(t) = m(t) x_1(t). \]

In such a case, the spectrum \( X(j\omega) \) is equal to the convolution of both factors spectra,

\[ X(j\omega) = \frac{1}{2\pi} M(j\omega) * X_p(j\omega). \]

When the primary component has the spectrum of the form (7), then

\[ X(j\omega) = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} X_n M(j\omega - jn\omega_1). \]

Thus, the spectrum is continuous, but lumped around harmonic frequencies, \( n\omega_1 \), in such a way that it can be confined by a profile as shown in Fig. 2.

The components with the frequency in the vicinity of harmonic frequencies, \( n\omega_1 \), resemble harmonics, but they lack the main property of harmonics. Namely, their frequency is not an integer multiplicity of the
Semi-periodic quantities are, however, usually much more complex. They cannot be described in terms of additive components or multiplicative factors separately. Quasi-harmonics are usually accompanied by interharmonic noise. The spectrum of such a semi-periodic quantity can be approximated by a profile as shown in Fig. 3. The symbol \( N \) denotes the level of the noise spectrum and the \( H_n \) denotes the width of the spectrum profile around frequency \( n\omega_1 \). A semi-periodic quantity approaches a periodic one when the values of \( N \) and \( H_n \) approach the zero value. The interharmonic noise spectrum around frequency \( n\omega_1 \), changes in general with frequency, so that \( N = N(\omega) \). Also, the frequency band \( H_n \) of quasi-harmonics can change with the order \( n \). Thus, \( N(\omega) \) and \( H_n \) can be used to characterize semi-periodic quantities.

Fig. 2. Spectrum profile of a modulated periodic quantity.

Particular areas of the spectrum profile shown in Fig. 3 can be associated with different properties of semi-periodic quantities. An increase in the peak value variation of the quantity broadens the spectrum around harmonic frequencies, \( n\omega_1 \), with little contribution to the interharmonic noise.

Semi-periodic quantities may differ substantially respective this interharmonic noise. They may contain a low frequency noise, below the primary frequency, \( \omega_1 \), a noise in a harmonic band of frequency and a high frequency noise or, at the same time, a noise in all three bands of frequency.

Apart from the situations described above, the semi-periodic currents can have the form of short, random pulses of a few cycles of the 60 or 50 Hz frequency. Welders are typical loads that produce such currents. Each pulse could be considered separately as a current of a finite energy [2].

6 Semi-periodic current orthogonal components

Although the period \( T \) of the primary component is not the period of a semi-periodic supply current, it can be used as a main time reference for power phenomena identification in circuits with such voltages and currents. Since this is the period of the generated voltage, a permanent flow of energy from generators to loads in a steady state of the circuit is associated only with the voltage and current components that have the period \( T \). The current components of a frequency different than the primary frequency, contribute to energy flow with a zero resultant value.

The voltages and currents observed over a single interval \( T \) in a cross-section between the supply and the load can be used for a periodic extension with the period \( T \). This hypothetical extension is periodic, thus eqs. (2), (3) and (5) can be calculated. However, if the voltages and currents are non-periodic, the values of these functionals will change in time. In particular, for a three-phase, three-wire system with line voltages and line currents vectors, \( u^T = [u_R, u_S, u_T] \) and \( i^T = [i_R, i_S, i_T] \), the scalar product

\[
\frac{1}{T} \int_T^{t+T} u^T i \, dt = \hat{P},
\]

provides the active power of the load in the observed interval \([t, t+T]\). It is referred to [6] as a “running active power” of the load. The symbol used emphasizes the variance of this power. Similarly,

\[
\sqrt{\frac{1}{T} \int_T^{t+T} u^T u \, dt} = \|u\|,
\]

is the supply voltage RMS value in the same interval. In practical situations the supply voltage distortion and asymmetry is much lower than distortion and asymmetry of the load voltage, therefore, it can be assumed in such situations that the supply voltage is sinusoidal and symmetrical. In such a case, \( \|u\| = \sqrt{3} U \), where \( U \) is the RMS value of the line-to-ground supply voltage.

The time-variant load is equivalent with respect to the active power in the interval \([t, t+T]\) at the voltage \( u \) to a resistive and balanced load of admittance

\[
\tilde{G}_e = \frac{\hat{P}}{\|u\|^2},
\]

referred [6] to as a “running equivalent conductance” of the load. Such equivalent resistive load draws the active current from the supply

\[
i_a = \tilde{G}_e u,
\]

of the “running RMS” value.
\[ \| \mathbf{\tilde{I}} \| = \frac{\tilde{P}}{\| \mathbf{u} \|}. \] (16)

The remaining part of the supply current
\[ i - i_u = i_r, \] (17)
consists of the reactive, \( i_r \), and unbalanced, \( i_u \), currents which are of the supply voltage frequency, that means primary currents, and secondary a current, \( i_s \), composed of quasi-harmonics and interharmonic noise, namely
\[ i_t = i_r + i_u + i_s. \] (18)

Thus, the load current can be decomposed into
\[ i = i_u + i_t + i_u + i_s. \] (19)

The reactive, unbalanced and secondary currents are useless and occur due to different phenomena. The reactive current is caused by the phase shift at the fundamental frequency, the unbalanced current is caused by the line currents asymmetry. The secondary current is an effect of quasi-harmonics and interharmonic noise. These four currents are mutually orthogonal, thus their running RMS values fulfill the relationship
\[ \| \mathbf{\tilde{I}} \|^2 = \| \mathbf{\tilde{I}}_r \|^2 + \| \mathbf{\tilde{I}}_u \|^2 + \| \mathbf{\tilde{I}}_u \|^2 + \| \mathbf{\tilde{I}}_s \|^2. \] (20)

The RMS values of the active, reactive and unbalanced currents are time-variant and they are equal to
\[ \| \mathbf{\tilde{I}}_r \| = \tilde{G}_c |\mathbf{u}|, \] (21)
\[ \| \mathbf{\tilde{I}}_u \| = |\tilde{B}_c| |\mathbf{u}|, \] (22)
\[ \| \mathbf{\tilde{I}}_u \| = |\tilde{A}| |\mathbf{u}|, \] (23)
where \( \tilde{B}_c \), and \( \tilde{A} \) are the running equivalent susceptance and unbalanced admittance of the load, respectively. To calculate these parameters and consequently, the RMS values of the supply current component, the equivalent line-to-line admittances of the load has to be calculated. This requires that the complex RMS (CRMS) value of the fundamental harmonic of two line currents, that means, for the line R
\[ \tilde{I}_{1R} = \frac{\sqrt{2}}{T} \int_0^{T} i_R(t) e^{-j\omega_0 t} dt, \] (24)
and similarly for the line S, as well as for the line-to-line voltages, \( u_{RT} \) and \( u_{ST} \) are calculated. These CRMS values can be obtained using the FFT algorithm on the current and voltage samples. Having these four running CRMS values, the equivalent line-to-line admittances of the load can be [7] obtained, namely
\[ \tilde{Y}_{RT1} = \frac{\tilde{I}_{1R}}{U_{RT1}}, \quad \tilde{Y}_{ST1} = \frac{\tilde{I}_{1S}}{U_{ST1}}. \] (25)

The running equivalent conductance, susceptance and unbalanced admittance of the load can be expressed in terms of line-to-line equivalent admittances as follows
\[ \tilde{G}_c = \text{Re}\left\{ \tilde{Y}_{RT1} + \tilde{Y}_{ST1} \right\}, \] (26)
\[ \tilde{B}_c = \text{Im}\left\{ \tilde{Y}_{RT1} + \tilde{Y}_{ST1} \right\}, \] (27)
\[ \tilde{A} = -\left( \tilde{Y}_{ST1} + \alpha \tilde{Y}_{RT1} \right), \] (28)
where \( \alpha = 1/e^{2\pi/3} \). The RMS value of the secondary component, \( \tilde{I}_s \), can be calculated from
\[ \| \mathbf{\tilde{I}}_s \|^2 = \| \mathbf{\tilde{I}}_s \|^2 - \left( \| \mathbf{\tilde{I}}_r \|^2 + \| \mathbf{\tilde{I}}_u \|^2 + \| \mathbf{\tilde{I}}_s \|^2 \right). \] (29)

The active current running RMS value and the load running equivalent conductance are functions of time. Therefore, they are not convenient measures of the power property of the load. It may be advantageous to define a conductance
\[ \tilde{G}_c = \frac{1}{M} \sum_{m=1}^{M} \tilde{G}_{c,m}, \] (30)
averaged over \( m = 1, 2, 3 \ldots M \), cycles of the voltage. When \( M \) approaches infinity, this averaged conductance approaches a constant value and the load with such conductance draws the active current
\[ i_{aa} = \tilde{G}_c u. \] (31)
of a constant RMS value.

Although its RMS value is affected by the averaging interval, \( MT \), it could be referred to as a “permanent component” of the active current. The difference
\[ i_u - i_{aa} = \left( \tilde{G} - \tilde{G}_c \right) u = i_{av}. \] (33)
is not associated with the permanent transmission of energy from the supply to the load. It could be referred to as a “fluctuating component” of the active current.

Eqs. (19) and (33) can be combined into the following decomposition of the supply current
\[ i = i_{aa} + i_{av} + i_r + i_u + i_s, \] (34)
In this decomposition only the averaged current \( i_{aa} \) is really useful and necessary for providing energy to the load. The current components \( i_{avr}, i_{av}, i_r, i_u \) and \( i_s \) do not only have not a distinctively different physical meaning but provide an insight into possibility of their compensation. The reactive current \( i_r \) can be reduced by a compensator of the reactive power. It could be a reactive or a switching compensator. A balancing compensator is needed for reducing the unbalanced current \( i_u \). It can also be built as a reactive or as a switching compensator. However, a switching compensator is required for reducing the secondary current, \( i_s \). A similar compensator is needed for reducing the variable component of the active current, \( i_{av} \), but with much more stringent requirements as to capability of energy stor-
age than the switching compensator needed for compensation of the remaining components.

7 Conclusions

Non-periodic currents exhibit a great variety of features and it is difficult to handle them in a unified way. There are very few studies on their power properties which are much more sophisticated than the power properties of periodic currents. This paper demonstrates, however, that the power theory developed by the author [8] for three-phase systems with periodic currents can be adopted for systems with semi-periodic currents.

8 List of symbols

\(x(t), |x|\) \small{instantaneous and RMS value of quantity \(x\)}  
\(X(j\omega)\) \small{quantity \(x\) spectrum}  
\(n\) \small{harmonic order}  
\(X_n\) \small{complex RMS (CRMS) value of \(n\) order harmonic}  
\(\omega_1, T\) \small{fundamental radial frequency and fundamental period}  
\(f_p, T_p\) \small{primary frequency and primary period}  
\(x_p, x_s\) \small{primary and secondary components of quantity \(x\)}  
\(\delta(\omega)\) \small{Dirac’s delta impulse in frequency domain}  
\(m(t), M(j\omega)\) \small{modulating function and its spectrum}  
\(N\) \small{level of the interharmonic noise spectrum}  
\(H_n\) \small{width of the spectrum profile around frequency \(n\omega_1\)}  
\(u, i\) \small{vectors of three-phase voltages and line currents}  
\(\tilde{P}, |\tilde{P}|\) \small{running active power and RMS value}  
\(\tilde{Y}, \tilde{A}\) \small{running admittance and unbalanced admittance}  
\(\tilde{G}_c, \tilde{B}_c\) \small{running equivalent conductance and susceptance}  
\(i_a, |i_a|\) \small{active current and its running RMS value}  
\(i_u, |i_u|\) \small{unbalanced current and its running RMS value}  
\(i_r, |i_r|\) \small{reactive current and its running RMS value}  
\(i_s, |i_s|\) \small{secondary current and its running RMS value}  
\(\tilde{G}_{em}, \tilde{G}_e\) \small{equivalent running conductance over cycle \(m\) and its averaged value}  
\(i_{aa}, i_{av}\) \small{permanent and fluctuating component of the active current}

References


Acknowledgement

This research was sponsored by the National Science Foundation, USA, under the Grant ESC-9810167.

Manuscript received on 

The Author

Leszek S. Czarnecki (1939) received the M.Sc. and Ph.D. degrees in electrical engineering and Habil. Ph.D. degree from the Silesian Technical University, Poland, in 1963, 1969 and 1984, respectively, where he was employed as an Assistant Professor. Beginning in 1984 he worked for two years at the Power Engineering Section, Division of Electrical Engineering, National Research Council of Canada as a Research Officer. In 1987 he joined the Electrical Engineering Dept. at Zielona Gora Technical University, Poland. In 1989 Dr. Czarnecki joined the Electrical and Computer Engineering Dept. of Louisiana State University, Baton Rouge, where he is a Professor of Electrical Engineering now. For developing a power theory of three-phase nonsinusoidal unbalanced systems and methods of compensation of such systems he was elected to the grade of Fellow IEEE in 1996. His research interests include network analysis and synthesis, power phenomena in nonsinusoidal systems, compensation and supply quality improvement in such systems.

E-mail: lsczar@communique.net  
Home page: http://www.geocities.com/CapeCanaveral/4739